U.S. Risk and Treasury Convenience*

Giancarlo Corsetti[†] Simon Lloyd[‡] Emile Marin[§] Daniel Ostry[¶]

September 11, 2026

Abstract

We document large, persistent shifts in investors' assessment of U.S. risk relative to other advanced economies since the late 1990s, driven by a rise in permanent risk but not reflected in currency returns. We investigate this disconnect using a two-country, incomplete-markets framework with trade in risky assets, currencies, and a rich maturity structure of bonds which earn convenience yields. We estimate an equilibrium relationship among relative country risk, *cross-border* convenience yields, and currency returns. Our estimates reveal that the rise in relative permanent risk explains 68% and 49% of the fitted decline in long-maturity convenience yields over the periods 2002-2006 and 2010-2014, respectively.

Key Words: Convenience Yields; Exchange Rates; Long-Run Risk; U.S. Safety, Equity Risk Premium.
JEL Codes: F30; F31; G12.

*We are particularly grateful to our discussants, Riccardo Colacito, William Diamond, Zhengyang Jiang, Iryna Kaminska and Wenhao Li, for their constructive suggestions. We also thank Vasco Carvalho, Mikhail Chernov, Pierre De Leo, Wenxin Du, Luca Fornaro, Oleg Itskhoki, Òscar Jordà, Arvind Krishnamurthy, Karen Lewis, Ian Martin, Fabrizio Perri, Hélène Rey, Liliana Varela, Rosen Valchev, Adrien Verdelhan, Steve Wu and presentation attendees at the BIS-BoE-ECB-IMF Conference on 'New Global Challenges Amid Incomplete and Divergent Recoveries', Federal Reserve Board, International Association for Applied Econometrics Annual Conference 2022, Tilburg University, PSE Macro Days 2022, University of Cambridge, Banque de France Workshop on Financial Frictions in International Macroeconomics, Bank of England, CEBRA Annual Meeting 2023, NBER Summer Institute 2023, Venice Finance Workshop 2023, SNB-FRB-BIS Third High-Level Conference on Global Risk, Uncertainty, and Volatility, EUI, Queen Mary University of London, IAAE 2025 the Bristol Finance Workshop, the Jackson Hole International Finance Workshop 2026 and SED 2026 for helpful comments. This paper is part of the project 'Disaster Risk, Asset Prices and the Macroeconomy' (JHUX) sponsored by the Keynes Fund at the University of Cambridge. Any views expressed are solely those of the authors and so cannot be taken to represent those of the Bank of England or Bloomberg LP. This paper should therefore not be reported as representing the views of the Bank of England or members of the Monetary Policy Committee, Financial Policy Committee or Prudential Regulation Committee.

[†]European University Institute and C.E.P.R. Email Address: `giancarlo.corsetti@eui.eu`.
[‡]Bloomberg LP and Centre for Macroeconomics. Email Address: `splloyd@gmail.com`.
[§]University of California, Davis. Email Address: `emarin@ucdavis.edu`.
[¶]Bank of England and Centre for Macroeconomics. Email Address: `daniel.ostry@bankofengland.co.uk`.

1 Introduction

Is the U.S. still a safe haven, and, if not, what does this mean for U.S. debt, equity returns, and the U.S. dollar (USD)? While the USD has generally retained its strength over recent decades, with no secular change in currency portfolio returns, during the same period and especially since the Global Financial Crisis (GFC), U.S. equities have outperformed the rest of the world. A significant share of these equity returns was anticipated (Figure 1)– consistent with classical explanations of compensation for risk, as opposed to noise or 'good luck'. Yet, insulation of currency returns from risk is inconsistent with the predictions of a large class of models which assume that markets are complete and integrated (Backus et al., 2001; Engel, 2014; Lustig et al., 2019). These models represent a special case of the following general no-arbitrage relationship:

$$\text{Currency portfolio returns} = \text{Cross-country risk differential} + \text{Complete-markets deviation} \tag{1}$$

If markets are complete and integrated, movements in currency portfolio returns should precisely mirror changes in cross-country risk differentials. However, we show that, over the 1997–2021 period, sizable fluctuations in risk differentials were not reflected in currency returns (Figure 2). As such, deviations from complete markets are required for theory to be reconciled with data.

In this paper, we consider a two-country, no-arbitrage model with internationally incomplete markets and convenience yields to study equity, bond and currency markets jointly. It is well understood that investors have long been willing to forgo other higher yield, risk-free government bonds in favor of 'convenient' U.S. Treasuries, as measured by deviations from covered interest parity (CIP) (Du et al., 2018a). While these convenience yields have remained positive at short maturities (at least until the end of our sample), long-maturity U.S. Treasuries appear to have progressively 'lost their shine'. Since the early 2000s, CIP deviations on long-maturity U.S. bonds have fallen, and, since 2010, U.S. Treasuries have frequently traded at a discount relative to many advanced-economy currencies (Figure 3).

Through the lens of the model, a U.S. investor long in Foreign bonds can be compensated for volatility in their stochastic discount factor (SDF) either via *pecuniary* returns on their currency portfolio, specifically driven by the dollar factor (Lustig et al., 2011); or via *non-pecuniary* convenience yields on Foreign bonds, reflected by a fall in CIP deviations. In our model, the stability of *average* dollar-trade returns in the data implies that a rise in relative U.S. risk and a fall in convenience yields on *long-maturity* U.S. Treasuries are two sides of the same coin. Both reflect a deterioration of investors' perceptions of relative U.S. *permanent* risk, which has not been reflected (so far) in the dynamics of the USD or bond premia.

Building on an established tradition in finance (e.g., Hansen and Jagannathan, 1991), we construct empirical measures of risk for a panel of advanced economies. Rather than relying on realized equity returns, we construct valuation-based measures of *ex ante* equity risk premia to remove noise and better isolate compensation for risk (Campbell and Thompson, 2007; Campbell, 2008).[1] A key driver of our measure is the expected growth of payouts (De La'O and Myers, 2021; Atkeson et al., 2024), which we construct from past payouts (dividends and buybacks) and survey forecasts of output. Our measure for the U.S. increases persistently relative to other advanced economies, although with substantial cyclical swings; because the expected volatility of equity returns does not move commensurately, these movements reflect changes in risk. Almost the entirety of this variation is concentrated in the permanent component of risk (Alvarez and Jermann, 2005)—consistent with explanations relying on revisions to the perceived probability of disaster (Farhi and Gourio, 2018; Kozlowski et al., 2019).[2]

To bring condition (1) to the data, we solve for the complete-markets deviation as a function of both convenience yields and risk differentials, deriving an equilibrium relationship in which U.S. and Foreign risk may enter with asymmetric weights. We estimate this relationship for 10-year maturity assets over 6-month holding periods using dynamic OLS (Saikkonen, 1991; Stock and Watson, 1993) and identify a long-run relationship in which permanent-risk differentials load negatively on long-maturity CIP deviations (our proxy for convenience yields): a 1p.p. increase in relative U.S. permanent risk is associated with approximately a 1.3b.p. decline in 10-year CIP deviations. Cyclical variation in relative risk also tracks contemporaneous movements in convenience yields: a 1p.p. innovation in the permanent-risk differential is associated with a 0.39b.p. fall in long-maturity convenience yields. These associations are not mechanical. Condition (1) is a no-arbitrage restriction that places no constraint on *which* margin absorbs the

[1]The fact that 'good luck' cannot explain the high equity premium in the U.S. concurs with earlier work by Constantinides (2002) and Brandt et al. (2006) who argue that 'good luck' can only explain about 30% over a longer sample.

[2]Permanent risk refers to the volatility of innovations which affect investors' valuations of returns in the long run. Consistent with this interpretation, our measure of U.S. (and other countries') permanent risk falls sharply *during* the dot-com crash and the GFC, as a disaster is realized rather than anticipated (see, e.g., Caramp and Silva, 2025), and is higher in their aftermath.

risk differential—pecuniary returns, measured convenience yields, or the unobserved deviation term—nor on the *magnitude* of the response. Moreover, the two series are constructed from largely distinct inputs: relative permanent risk from equity valuations and payout forecasts, convenience yields from currency forwards and swaps.

Our estimated coefficients are robust to including intermediary leverage (He et al., 2017), relative debt-to-GDP, alternative risk measures such as the VIX, and survey-based currency returns. Nonetheless, we find that the relative debt-to-GDP and the inverse intermediary capital ratio are both significant predictors of long-maturity convenience yields, operating through complementary economic channels. Lower U.S. debt and higher intermediary leverage each imply an increase in the relative convenience yield on U.S. bonds, consistent with theory.

To illustrate the economic significance of these findings, we use the coefficients estimated over the full sample to construct counterfactual paths for 10-year CIP deviations under scenarios in which relative permanent-risk differentials remain constant. In our exercises, we fix U.S. permanent risk *vis-à-vis* the other advanced economies near its troughs during the dot-com bubble and the GFC, respectively, letting other variables evolve as in the data. We find that, over the periods 2002–2006 and 2010–2014, a rise in relative permanent risk explains approximately 68% and 49%, respectively, of the fall in long-maturity convenience yields explained by our model—in both cases corresponding to ≈55% of the actual decline. While we report several goodness of fit measures which evaluate the model over the entire sample, the historical counterfactual shares evaluate the mechanism precisely during episodes when relative risk moves substantially.

Finally, we show that the long-maturity equilibrium relationship contrasts starkly with the link between risk, returns, and convenience at short maturities. While short-maturity dollar-trade returns, though volatile, are associated with short-maturity convenience yields (Engel and Wu, 2023; Jiang et al., 2021a), we find no significant role for our measured risk differentials. Motivated by this result, we discuss a parsimonious extension of our theoretical framework allowing for segmentation between markets for short- and long-lived assets.[3] With segmented markets, the USD can remain insulated in the near term, despite heightened uncertainty about relative U.S. permanent risk.

[3] In this environment, the risk proxies we construct using the dividend-discount model are appropriate measures of the SDFs pricing long-maturity assets, but other pricing factors are likely to confound our results at short maturities (see Du et al., 2023).

Related Literature. Our paper lies at the intersection of international macroeconomics, which has traditionally focused on the Backus and Smith (1993); Kollmann (1995) condition, and international finance which has focused on deviations from the uncovered interest parity (UIP). The no-arbitrage condition (1) we focus on integrates both perspectives: unlike the former, it does not rely on consumption data, and unlike the latter, it explicitly highlights the role of risk (SDF volatility). Our starting point is the risk-based determination of exchange-rate returns (Backus et al., 2001; Verdelhan, 2010), extended to account for convenience yields (Jiang et al., 2021a,b, 2024; Engel and Wu, 2023).[4] Our work also draws closely on models with incomplete financial markets (Pavlova and Rigobon, 2007, 2008; Corsetti et al., 2008; Lustig and Verdelhan, 2019, amongst others), where risk premia arise endogenously with no-arbitrage. Relative to these papers, our contribution is to bring the equilibrium relationship to the data.[5]

We shed new light on widely-studied asymmetries in the international monetary system. The U.S. has historically benefited from an excess return on external assets (e.g., Gourinchas et al., 2010), seigniorage from abroad (e.g., Du et al., 2018a; Jiang et al., 2021a, 2024), but experiences currency appreciation in global downturns as investors 'fly to safety' (e.g., Maggiori, 2017; Arvai and Coimbra, 2024; Kekre and Lenel, 2024). We emphasize that, as relative U.S. risk has increased over recent decades, convenience yields on long-maturity U.S. Treasuries have declined, yet the U.S. dollar has remained insulated.

The literature on convenience yields focuses on their measurement and drivers, such as liquidity, safety, and limits to arbitrage (Krishnamurthy and Vissing-Jorgensen, 2012; Du et al., 2018a,b; Liu et al., 2020; Diamond and Van Tassel, 2022; Acharya and Laarits, 2024; Augustin et al., 2024; Liao and Zhang, 2025). Relatedly, Bianchi et al. (2021) and Bianchi and Bigio (2026) investigate endogenous convenience yields arising from banks' precautionary demand for dollar liquidity under settlement frictions. Other studies use convenience to explain exchange-rate dynamics, predominantly at short horizons (Krishnamurthy and Lustig, 2019; Valchev, 2020; Jiang et al., 2021a,b; Engel and Wu, 2023). We focus on the fall in *long-maturity* U.S. Treasury convenience. Taking the microstructure view, Jermann (2020), He et al. (2022) and

[4] Complementary explanations rely on limits to arbitrage (Jeanne and Rose, 2002; Gabaix and Maggiori, 2015; Vayanos and Vila, 2021; Gourinchas et al., 2025), long-run risk (Colacito and Croce, 2011), disaster risk (Farhi and Gabaix, 2016) and transitory risk (Lloyd and Marin, 2020), among others.

[5] Methodologically, we draw on a large literature using asset prices, specifically equity, to ascertain characteristics of SDFs (Hansen and Jagannathan, 1991; Alvarez and Jermann, 2005; Hansen and Scheinkman, 2009; Bakshi and Chabi-Yo, 2012; Chabi-Yo and Colacito, 2019) and macro risk (Farhi and Gourio, 2018; Atkeson et al., 2024, 2025; Greenwald et al., 2025).

Du et al. (2023) investigate the drivers of this fall *within* the U.S, while Du et al. (2026) highlight the decoupling between cross-country convenience and within country convenience. Our paper proposes a complementary explanation, drawing a link between macroeconomic risk and cross-country convenience yields.

The remainder of this paper is structured as follows. Section 2 presents key stylized facts on equity, dollar-trade returns, and the Treasury basis. Section 3 details the theoretical framework and derives the equilibrium relationships. Section 4 discusses the mapping of theory to the data. Section 5 presents our empirical analysis. Section 6 concludes.

2 Stylized Facts on Equities, Currencies and the Treasury Basis

We motivate our analysis with three stylized facts on equity returns, currency returns and the Treasury basis. Later in the paper, we use the data shown here to derive theory-consistent measures to estimate no-arbitrage condition (1). Throughout, we focus on seven advanced-economy (AE) currencies: USD as the base against AUD, CAD, CHF, EUR, GBP and JPY, from the late 1990s to 2021:03, and defer a detailed description of the data to Section 4.

#1. Large Swings in U.S. vs. AE Expected Equity Premia. U.S. equities have generally outperformed other AE markets over our sample. While some of these returns were unexpected (i.e., 'good luck'), a large component was anticipated and served as compensation for relatively higher U.S. risk. To capture this, we rely on valuation-based pricing (Campbell and Thompson, 2007; Campbell, 2008) and construct expected equity risk premia building on restrictions from widely-used steady-state valuation models (e.g., Gordon, 1962).

Expected (log) equity risk premia $R^{eq}_{t,t+1}/R_t$ can be constructed as:[6]

$$\log \mathbb{E}_t\left[\frac{R^{eq}_{t+1}}{R_t}\right] = \log\left(1 + \frac{TCF_t}{P_t} + g^e_t\right) - \log\left(R_t\right) \tag{2}$$

[6]Consider the simple static growth model in steady state:

$$\frac{P_t}{TCF_t} = \frac{1}{(1+r_t) + \mathbb{E}_t[ERP_t] - (1+g^e_t)}.$$

Define $\mathbb{E}_t[R^{eq}_t] = (1+r_t) + \mathbb{E}_t[ERP_t]$, substitute this into the previous expression, and rearrange to yield: $\mathbb{E}[R^{eq}_t] = 1 + \frac{TCF_t}{P_t} + g^e_t$. Using $\log \mathbb{E}_t\left[\frac{R^{eq}_t}{R_t}\right] = \log \mathbb{E}_t[R^{eq}_t] - \log(1+r_t)$ delivers equation (2). In related work, Marin and Jung (2026) estimate expected equity returns without relying on steady-state restrictions, building on Greenwald et al. (2025), and find a qualitatively similar pattern.

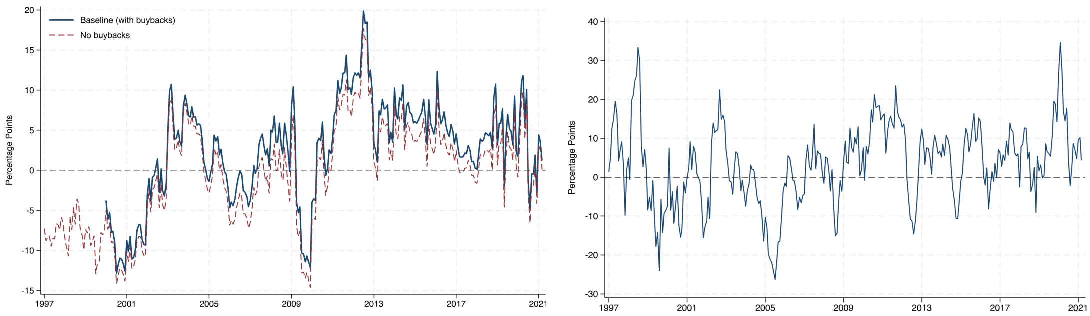
where $R^{eq}_{t,t+1}$ denotes the gross return to equities, R_t denotes the gross one-period risk-free rate, TCF_t/P_t denotes the total cash flow (payout) to price ratio and g^e_t is the *expected* net growth rate of payouts at time t. For the U.S., Europe, and the U.K., where buybacks are prevalent and data is available, $\frac{TCF_t}{P_t} = \frac{D_t}{P_t} + \frac{B_t}{P_t}$, where D_t/P_t denotes the dividend yield on the country's broad equity index and B_t/P_t is the analogous net buyback yield, computed as the ratio of monthly net repurchases to market capitalization. For Australia, Canada, Switzerland, and Japan, where a comparable net-buyback series is not available, we set $\frac{TCF_t}{P_t} = \frac{D_t}{P_t}$. The expected growth rate g^e_t is the fitted value from a country-specific predictive regression of realized 6-month-ahead dividend growth on the contemporaneous dividend yield, lagged dividend-growth realizations, and one-year-ahead forecasts of nominal GDP growth from *Consensus Economics*. The forecasting regression delivers an adjusted R^2 of 46% for the U.S. (Table B.3), with lagged dividend growth and one-year-ahead GDP growth expectations the strongest predictors.

Investigating our measure, we find that time variation in cross-country expected returns is predominantly driven by variation in expected payout growth g^e_t (Figures B.3 and B.4, Appendix B.2). In our baseline specification, we assume a constant payout ratio, so that investors' expected growth in buybacks is equal to the expected growth in dividends. The resulting payout growth measure is strongly correlated (73%) with the analyst-based dividend-growth expectations used by De La'O and Myers (2021), available for the U.S. only. Our results are nonetheless robust to assuming buybacks are completely unexpected (Table B.7, Appendix B.5).

Figure 1(a) plots the annualized expected (*ex ante*) equity premia for the U.S. *relative* to other AEs. This displays large swings with distinct peaks in U.S. relative 'risk' in 2003 and 2012, where the difference in annualized premia reaches 11p.p. and 19p.p., respectively. Conversely, the U.S. was relatively 'safe' following financial crises (in 2001 and 2009), in line with a large literature characterizing the U.S. as an insurer during global downturns (Gourinchas et al., 2010; Maggiori, 2017). Notably, between end-2009 and mid-2012, in the aftermath of the GFC, there was a trough-to-peak increase in relative U.S. premia of over 30p.p. Moreover, by the end of the sample, relative U.S. premia were well above their late-1990s level. Even though the trend increase in equity risk premia we estimate is modest (0.48p.p. per year, $t = 4.4$), small changes in these premia can be enough to explain large fluctuations in convenience yields due to differences in the scale of equity return differentials (2.2p.p. average over the entire sample) *vis-*

Figure 1: U.S. vs. Advanced Economy Expected and Realized Equity Risk Premium

(a) Relative U.S. *expected* equity premium (b) Relative U.S. *realized* equity premium



Note. U.S. net average AE *expected* (Panel a) and *realized* (Panel b) 6-month equity excess returns. Expected returns are constructed from equation (2), with expected dividend growth estimated as in equation (B.1) (Appendix B.2). Sample: 1997:01–2021:03.

à-vis convenience yields (≈10b.p.). For comparison, Figure 1(b) plots relative realized (*ex post*) equity returns which are more volatile and, on average, have a slightly higher mean (3.1p.p).

#2. No Significant Change in U.S. Dollar Returns. Next, we analyze the pecuniary return investors earn on portfolio funded in USD and long Foreign bonds of various (monthly) maturities k. The expected six-month return on a k-maturity bond, $\mathbb{E}_t[\mathrm{rx}_{t+1}^{CT,(k)}]$, is comprised of a currency risk premium and the difference in domestic bond premia:

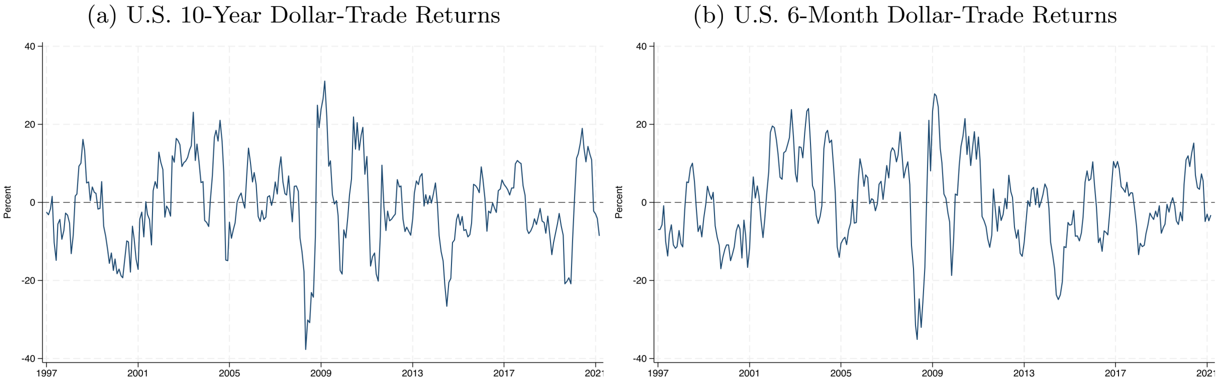
$$\mathbb{E}_t[rx_{t+1}^{CT,(k)}] := \underbrace{\mathbb{E}_t[rx_{t+1}^{FX}]}_{\text{Currency Returns}} + \underbrace{\mathbb{E}_t[rx_{t+1}^{(k)*}] - \mathbb{E}_t[rx_{t+1}^{(k)}]}_{\text{Difference in Local Bond Returns}} \tag{3}$$

where $rx_{t+1}^{FX} = r_t^* - r_t + \Delta e_{t+1}$, with $e_t = \log(\mathcal{E}_t)$ representing the (log) exchange rate, defined as the number of USD per unit of Foreign currency, and r_t^* representing the Foreign analog of r_t. Similarly, $rx_{t+1}^{(k)(*)}$ denotes the one-period log excess holding return on a k-maturity domestic bond, in the U.S. or the Foreign country (denoted with $*$).

Figure 2 plots annualized returns on long- (10-year, Panel (a)) and short-maturity (6-month, Panel (b)) dollar-trade returns, over 6-month holding periods. Over the sample, the mean 10-year dollar-trade return is statistically indistinguishable from zero ($t = -0.36$) and exhibits no time trend ($t = -0.48$), while its variance is virtually identical before and after the GFC. In contrast to relative risk, although volatile, currency returns display no secular movement.

So, if the large fluctuations and secular movements in relative U.S. expected equity premia (against other advanced economies) over the sample are indicative of increasing relative U.S.

Figure 2: U.S. Long- and Short-Maturity Dollar-Trade Returns



Note. One-period (6-month holding) dollar-trade returns on 10-year (Panel a) and 6-month (Panel b) bonds ($rx_{t+1}^{CT,(\infty)}$ in (3) and $rx_{t+1}^{FX} = r_t^* - r_t + \Delta e_{t+1}$ respectively) averaged across other advanced economy currencies *vis-à-vis* the USD, in percent per year. Sample: 1997:01–2021:03.

risk, but dollar-trade returns do not move to compensate, condition (1) suggests that deviations from the complete and frictionless international financial markets must be changing to restore no-arbitrage. For this, we turn to the Treasury basis constructed by Du et al. (2018a).

#3. Falling Treasury Basis on Long-Maturity Bonds. Define deviations from the k-month covered interest rate parity (CIP) condition as the (annualized) log return on a covered position (using an exchange-rate forward $f_t^{(k)}$), long on a k-period Foreign bond and short on a k-maturity U.S. Treasury (Home bond):

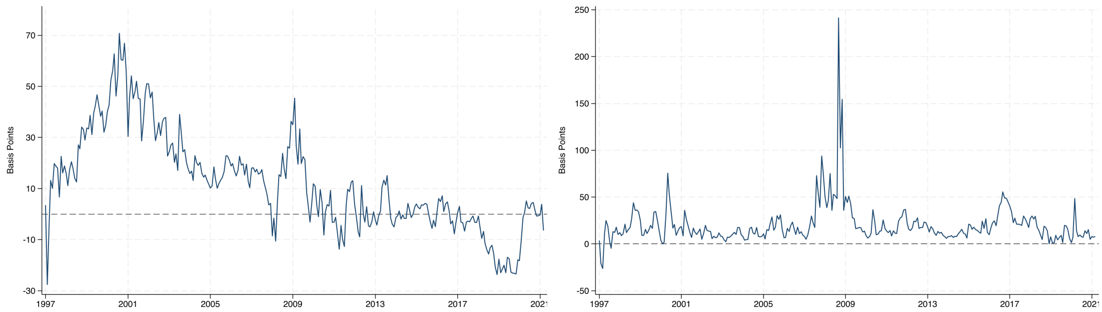
$$CIP_t^{(k)} := \underbrace{r_t^{(k)*} - r_t^{(k)}}_{\text{log interest rate differentials}} + \underbrace{f_t^{(k)} - e_t}_{\text{forward premium}} \tag{4}$$

When $CIP_t^{(k)} > 0$, the pecuniary return on a synthetic USD-denominated bond, $f_t^{(k)} - e_t + r_t^{(k)*}$, is greater than the return on a k-maturity U.S. Treasury, $r_t^{(k)}$. Arbitraging CIP deviations is riskless so Foreign investors must intrinsically prefer U.S. bonds—i.e., these bonds offer a relatively higher non-pecuniary return. While CIP deviations for all maturities are small relative to other premia, they are larger when mapped to convenience yields (in the utility space)—as shown by Jiang et al. (2021a) —which is the relevant benchmark for no-arbitrage condition (1).

Figure 3(a) documents a persistent decline in 10-year CIP deviations over our sample. Having been as high as 70b.p. in the early 2000s, long-maturity CIP deviations turned negative post-2016. Put differently, after 2016, foreign investors required compensation to hold long-

Figure 3: Long- and Short-Maturity U.S. Treasury Basis

(a) U.S. 10Y Treasury Basis (b) U.S. 6M Treasury Basis



Note. Panels 3a and 3b display time series of the long-maturity and short-maturity U.S. Treasury basis, respectively. These reflect CIP deviations (defined in (4)), where the government bond maturities are 10 years (long-maturity) and 6 months (short-maturity), respectively. In each case, the time series are a cross-sectional average across six advanced-economy currencies *vis-à-vis* the U.S. The sample runs from 1997:01 to 2021:03.

maturity Treasuries. As well as moving in the opposite direction to expected equity premia over our sample, the peaks and troughs of their cyclical fluctuations also broadly correspond.

The profile of short-maturity convenience yields in panel (b) is sharply different. 6-month deviations remain positive throughout and peaked much higher during the GFC (almost 250b.p).

3 Model of Risk, Returns and Convenience Yields

We next present a theoretical framework to study these empirical facts. Consider an environment with two countries: Home (the U.S.) and Foreign (denoted with an asterisk $*$). Representative investors in each country trade in zero-coupon bonds of varying maturities $k = 1, 2, ..., \infty$ issued in both economies. The bonds pay a known return in local currency at maturity and are free from default risk. Bonds also deliver a non-pecuniary convenience yield which is investor $\{i = H, F\}$, issuer $\{j = H, F\}$, and maturity k-specific, and varies over time. Investors also trade in risky assets which carry lower convenience, normalized to zero.

Throughout, we do not rely on a log-normal approximation and use entropy operators instead. This is because heteroskedastic models generate non-log-normal returns at longer maturities, even when short-maturity returns are log-normal (Campbell et al., 1997). For a generic variable X, conditional entropy is $\mathcal{L}_t(X_t) \equiv \log \mathbb{E}_t[X_t] - \mathbb{E}_t[\log X_t]$, where $\mathcal{L}_t(X_t) = \frac{1}{2}\text{var}_t(\log X_t)$ if X_t is log-normally distributed.[7] Conditional co-entropy, $\mathcal{C}_t(X_t, Y_t)$, a generalized notion of

[7]This measure of conditional volatility $\mathcal{L}_t(X)$, often referred to as Theil (1967)'s second conditional entropy measure, is in general equal to one half the second-order cumulant ($\frac{\kappa_{2,t}}{2!} = \frac{\text{var}_t(X_{t+1})}{2}$) plus all higher-order cumulants ($\frac{\kappa_{3,t}}{3!} + \frac{\kappa_{4,t}}{4!} + ...$). If $\text{var}_t(X_t) = 0$, then $\mathcal{L}_t(X_t) = 0$.

covariance discussed in Backus et al. (2018), is analogously defined as $\mathcal{C}_t(X_t, Y_t) = \mathcal{L}_t(X_t Y_t) - \mathcal{L}_t(X_t) - \mathcal{L}_t(Y_t)$.

3.1 Asset Markets

3.1.1 Bond Markets

Define $P_t^{(k)}$ as the date-t price of a Home zero-coupon bond of maturity k. The gross pecuniary return on this bond, earned at maturity but known at time t, is: $R_t^{(k)} = 1/P_t^{(k)}$. The 'risk-free rate' R_t is defined for $k = 1$, such that: $R_t \equiv R_t^{(1)} = 1/P_t^{(1)}$. We define $R_{t,t+1}^{(k)(*)}$ as the gross return from purchasing a k-period bond at time t and selling it at $t+1$ as a $k-1$-period bond. An investor i purchasing a country-j bond at time t that is held to maturity k earns a convenience yield $\theta_t^{i,j(k)}$, as formalized below.[8]

Assumption 1 (Convenience-Yield Term Structure) *Home and Foreign investors trade in Home and Foreign risk-free bonds of maturity $k = 1, 2, ..., \infty$. The term structure of convenience yields held to maturity (i.e., $\theta_t^{i,j(k)}$ for an investor i purchasing a k-period country-j bond at time t) is observable at time t.*

Denote the Home (Foreign) period-t nominal pricing kernel by Λ_t (Λ_t^*). In turn, define the Home (Foreign) k-period SDF between periods t and $t+k$ as $M_{t,t+k} \equiv \Lambda_{t+k}/\Lambda_t$ ($M_{t,t+k}^* \equiv \Lambda_{t+k}^*/\Lambda_t^*$). With these SDFs, Euler equations for Home and Foreign agents investing in k-period Home and Foreign risk-free bonds are given by:

$$\mathbb{E}_t \left[M_{t,t+k}\right] R_t^{(k)} = e^{-\theta_t^{H,H(k)}} \tag{5}$$

$$\mathbb{E}_t \left[M_{t,t+k}^*\right] R_t^{(k)*} = e^{-\theta_t^{F,F(k)}} \tag{6}$$

$$\mathbb{E}_t \left[M_{t,t+k} \frac{\mathcal{E}_{t+k}}{\mathcal{E}_t}\right] R_t^{(k)*} = e^{-\theta_t^{H,F(k)}} \tag{7}$$

$$\mathbb{E}_t \left[M_{t,t+k}^* \frac{\mathcal{E}_t}{\mathcal{E}_{t+k}}\right] R_t^{(k)} = e^{-\theta_t^{F,H(k)}} \tag{8}$$

for all maturities k. The nominal exchange rate $\mathcal{E}_t$ represents the number of USD per unit of Foreign currency, so an increase in $\mathcal{E}_t$ represents a U.S. depreciation and a Foreign appreciation.

[8] To the extent that convenience yields arise from a bond's value as collateral, this timing assumption reflects that collateral value is accounted for in contracts written at time t. Convenience yields are often micro-founded using bond-in-utility formulations as in Valchev (2020), Jiang et al. (2021b), and Jiang and Richmond (2023).

3.1.2 Foreign Exchange

If (5)-(8) hold, any exchange-rate process which admits no arbitrage must satisfy:

$$\begin{aligned}\mathcal{L}_t(\mathcal{E}_{t+1}/\mathcal{E}_t) + \mathcal{L}_t(\mathcal{E}_t/\mathcal{E}_{t+1}) = &- \mathcal{C}_t(M_{t,t+1},\ \mathcal{E}_{t+1}/\mathcal{E}_t) - \mathcal{C}_t\big(M^*_{t,t+1},\ \mathcal{E}_t/\mathcal{E}_{t+1}\big) \\ &+ \left(\theta_t^{H,H(1)} - \theta_t^{H,F(1)}\right) - \left(\theta_t^{F,H(1)} - \theta_t^{F,F(1)}\right)\end{aligned} \tag{9}$$

We consider an exchange-rate process given by:

$$\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = \frac{M^*_{t,t+1}}{M_{t,t+1}} e^{\eta_{t+1}}, \tag{10}$$

where η_{t+1} is the incomplete-markets wedge (see, e.g., Backus et al., 2001; Lustig and Verdelhan, 2019). With complete markets, absent convenience yields, the condition is satisfied by setting $\eta_{t+1} = 0$. With incomplete markets, the wedge η_{t+1} depends on convenience-yield differentials. Combining (5) and (8), as well as (6) and (7), using (10) yields:

$$\mathbb{E}_t[\eta_{t+1}] = \mathcal{L}_t(e^{-\eta_{t+1}}) + \mathcal{C}_t(M_{t,t+1},\ e^{-\eta_{t+1}}) + \theta_t^{F,H(1)} - \theta_t^{H,H(1)}, \tag{11}$$

$$\mathbb{E}_t[\eta_{t+1}] = -\mathcal{L}_t(e^{\eta_{t+1}}) - \mathcal{C}_t(M^*_{t,t+1},\ e^{\eta_{t+1}}) + \theta_t^{F,F(1)} - \theta_t^{H,F(1)} \tag{12}$$

Ceteris paribus, an increase in the convenience yield enjoyed by Foreign investors $\theta_t^{F,H(1)}$, either leads to a Foreign-currency appreciation (through $\mathbb{E}_t[\eta_{t+1}]$), or must be compensated by making exchange-rate movements 'safer' for Home investors (Jiang et al., 2021b). In addition to (11) and (12), there is an analogous restriction on η_{t+1} for each long-maturity bond, as well as for each traded risky asset. The complete spanning limit corresponds to $\eta_{t+1} = \theta_t^{F,H(1)} - \theta_t^{H,H(1)} = \theta_t^{F,F(1)} - \theta_t^{H,F(1)}$.

3.1.3 Equity Markets

In addition to trading in domestic and foreign bonds, investors in each country trade in a domestic risky asset which we label equity.

Assumption 2 (Equities and Convenience) *Home and Foreign investors also trade in a respective domestic risky asset, with one-period return $R^g_{t,t+1}$ ($R^{g*}_{t,t+1}$), on which they derive a baseline level of convenience yield, normalized to zero.*

The returns on risky assets therefore satisfy the following Euler equations:

$$\mathbb{E}_t\left[M_{t,t+1}R^{g}_{t,t+1}\right] = 1, \tag{13}$$

$$\mathbb{E}_t\left[M^*_{t,t+1}R^{g*}_{t,t+1}\right] = 1 \tag{14}$$

3.2 Equilibrium Relationships

3.2.1 Short-Maturity

Define rx^{FX}_{t+1} as *ex post* (log) return from a one-period dollar-trade strategy that goes long the Foreign risk-free bond and short the US risk-free bond:

$$rx^{FX}_{t+1} = \log\left(\frac{R^*_t}{R_t}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) = r^*_t - r_t + \Delta e_{t+1} \tag{15}$$

Absent frictions, UIP holds ($\mathbb{E}_t[rx^{FX}_{t+1}] = 0$) if investors are risk neutral. UIP deviations ($\mathbb{E}_t[rx^{FX}_{t+1}] \neq 0$) thus reflect excess returns for which investors earn cross-border dollar-trade returns. In classic open-economy settings without convenience yields ($\theta^{i,j(k)}_t = 0$ for all i, j, k), 'risk-based' explanations of UIP failures draw a link between the covariance of investors' SDFs and returns on Foreign-currency portfolios (Backus et al., 2001).

When investors attach convenience yields to bonds, the link between the exchange-rate risk premium and relative SDF volatility (i.e., condition (1) for short-maturities) is detailed below:

Proposition 1 (SDF Volatility, FX Risk and Convenience at Short Maturities) *Given $M^*_{t,t+1}$, $M_{t,t+1}$ and the exchange-rate process (10), the expected excess return on a one-period currency portfolio, relative SDF volatility, and one-period convenience yields satisfy:*

$$\mathbb{E}_t[rx^{FX}_{t+1}] = \underbrace{\mathcal{L}_t(M_{t,t+1}) - \mathcal{L}_t(M^*_{t,t+1})}_{\text{Risk Differential}} + \underbrace{\theta^{F,H(1)}_t - \theta^{F,F(1)}_t}_{\text{Short-Maturity Convenience}} - \underbrace{\Omega^{(1)}_t}_{\text{IM contribution}} \tag{16}$$

where $\Omega^{(1)}_t = -\mathcal{L}_t(e^{-\eta_{t+1}}) - \mathcal{C}_t(M_{t,t+1},\, e^{-\eta_{t+1}})$ arises because markets are incomplete.

Proof: Combine expectations of the exchange-rate process (10) and log-entropy expansions of the $k = 1$-period domestic Euler equations (5) and (6). See Appendix A.2 for full derivation. $\square$

The equilibrium condition (16) holds in a large class of international macro models, including models with cross-country segmentation (Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021; Gourinchas et al., 2025). Suppose the Home investor faces greater SDF volatility ($\mathcal{L}_t(M_{t,t+1}) > \mathcal{L}_t(M^*_{t,t+1})$) and consider the complete-spanning limit ($\mathcal{L}_t(e^{\eta_{t+1}}) \to 0\ \forall t$). Then, by no arbitrage, they can be compensated either by greater expected pecuniary returns on a long position in the Foreign bond ($\mathbb{E}_t[rx^{FX}_{t+1}] > 0$) or greater non-pecuniary convenience yields on the Foreign bond relative to the Home bond ($\theta_t^{F,H(1)} - \theta_t^{F,F(1)} < 0$), or a combination of both. That is, in addition to the traditional open-economy logic that pecuniary currency returns compensate rational investors for bearing relative risk, non-pecuniary convenience yields provide a second channel through which asymmetries in SDF volatility across countries can equilibrate.

Away from the complete-spanning limit, investors can also be compensated by additional risk premia ($\Omega_t^{(1)}$), which make it difficult to take the equilibrium conditions to the data. As a first pass, we do not specify a functional form for the incomplete markets wedge (η_{t+1}), in order to illustrate all mechanisms at play. Then, Section 3.3 restricts attention to affine models and derives an explicit form of the equilibrium relationships, valid under both complete and incomplete international financial markets.

3.2.2 Transitory and Permanent Risk Decomposition with Convenience

To tie asset returns with risk and convenience at different horizons, we use the Alvarez and Jermann (2005) decomposition of pricing kernels. We decompose the Home pricing kernel Λ_t into two components:

$$\Lambda_t = \Lambda_t^{\mathbb{P}} \Lambda_t^{\mathbb{T}}, \tag{17}$$

where $\Lambda_t^{\mathbb{P}}$ is a martingale that captures the 'permanent' component of Λ_t, while $\Lambda_t^{\mathbb{T}}$ reflects the 'transitory' component. We decompose the Foreign pricing kernel analogously. Under regularity conditions, the permanent component is defined as $\mathbb{E}_t \Lambda_{t+1}^{\mathbb{P}} = \Lambda_t^{\mathbb{P}}$, where $\Lambda_t^{\mathbb{P}} = \lim_{k\to\infty} \frac{\mathbb{E}_t \Lambda_{t+k}}{\beta^{t+k}}$.[9] The permanent measure is unaffected by new information that does not lead to revisions of the expected value of Λ in the long run. The transitory component is defined by the residual $\Lambda_t^{\mathbb{T}} = \lim_{k\to\infty} \frac{\beta^{t+k}}{\mathbb{E}_t[\Lambda_{t+k}]/\Lambda_t}$ and is equivalent to a scaled long-term interest rate.

[9] Specifically, the decomposition assumes: (i) that there is a number β such that $0 < \lim_{k\to\infty} \frac{\mathbb{E}_t[\Lambda_{t+k}]/\Lambda_t}{\beta^k} < \infty$ for all t; and (ii) for each $t+1$ there is a random variable X_{t+1} such that $\frac{\Lambda_{t+1}}{\beta^{t+1}} \frac{\mathbb{E}_{t+1}[\Lambda_{t+1+k}]/\Lambda_{t+1}}{\beta^k} \le X_{t+1}$ almost surely, with $\mathbb{E}_t X_{t+1}$ finite for all k.

First, we highlight the link between the transitory component of representative investors' SDFs, long-maturity bond returns and long-maturity convenience yields:

Lemma 1 (Transitory SDF, Asset Prices and Convenience) *The transitory component of the Home representative investor's SDF is inversely related to the one-period holding return on an infinite-maturity bond (i.e., bond term premium), adjusted for the holding-period convenience yield:*

$$\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}} \equiv M_{t,t+1}^{\mathbb{T}} = \frac{1}{R_{t,t+1}^{(\infty)}} e^{-\theta_{t,t+1}^{H,H(\infty)}} \tag{18}$$

where $\theta_{t,t+1}^{H,H(\infty)} = \lim_{k\to\infty}\left(\theta_t^{H,H(k)} - \theta_{t+1}^{H,H(k-1)}\right)$ *and analogously for the Foreign investor.*

Proof: See Appendix A.3. □

Absent convenience yields, transitory innovations to pricing kernels reflect the bond term premium only. When there are convenience yields on domestic bonds, an investor holding an infinite-maturity bond for a single period earns the convenience yield today on that bond, but forgoes the residual convenience yield tomorrow. Holding-period convenience yields $\theta_{t,t+1}^{i,j(\infty)}$ are defined analogously for any i, j and play a vital role in subsequent analysis.

Second, we derive bounds on the volatility of the overall and permanent components of risk, in relation to equity premia. This is a key step, since it defines our preferred measure of overall and permanent risk for our empirical analysis.

Lemma 2 (Permanent and Total Risk, Asset Prices and Convenience) *The lower bound for conditional volatility of the Home investor's SDF in the presence of convenience yields is given by:*

$$\mathcal{L}_t(M_{t,t+1}) \geq \mathbb{E}_t \log\left[\frac{R_{t,t+1}^g}{R_t}\right] - \theta_t^{H,H(1)} \tag{19}$$

The lower bound on the conditional volatility of the permanent component of the Home representative investor's SDF is:

$$\mathcal{L}_t\left(M_{t,t+1}^{\mathbb{P}}\right) \geq \mathbb{E}_t \log\left[\frac{R_{t,t+1}^g}{R_t}\right] - \mathbb{E}_t\left[rx_{t+1}^{(\infty)}\right] - \mathbb{E}_t[\theta_{t,t+1}^{H,H(\infty)}] \tag{20}$$

where $rx_{t+1}^{(\infty)} = \log\left(R_{t,t+1}^{(\infty)}/R_t\right)$ *and analogously for the Foreign investor.*

Proof: See Appendix A.4. □

Inequality (19) bounds the conditional volatility of the Home investor's overall SDF—the measure of the country's overall riskiness—such that it is at least as large the expected (log) excess return on risky equities, net of the convenience yield the Home investor earns on the Home bond. While inequality (19) holds for any risky asset, the right-hand side is maximized by using the highest risk premium in the economy.[10]

Inequality (20) similarly bounds the permanent component of the Home investor's SDF—the measure of the country's permanent risk—such that it is at least as large as the difference between the expected (log) excess return on risky equities (the first term on the right-hand side of (20)) and the expected one-period return on the asymptotic discount bond, net of changes in the convenience yield on this bond. The expression isolates permanent risk because the bond premium and convenience yield on a long-maturity bond encodes the transitory (insurable) component of risk only. Holding-period convenience reflects that, by forgoing the one-period pecuniary return on an infinite maturity bond $\mathbb{E}_t\left[rx_{t+1}^{(\infty)}\right]$, the Home investor also forgoes today's convenience net of what they expect to recover tomorrow, captured by $\mathbb{E}_t\left[\theta_{t,t+1}^{H,H(\infty)}\right]$.

3.2.3 Long-Maturity

The failure to reject UIP empirically over long horizons (e.g., Chinn and Meredith, 2005; Lustig et al., 2019; Andrews et al., 2024), along with our first stylized fact, on cross-country risk differentials, imply that risk-based explanations impose counterfactually strict restrictions at long maturities. This, alongside our interest in permanent risk specifically, motivates our focus on long maturities. To derive an analog to Proposition 1 for long-maturity trades, we define two new objects. The first is the one-period dollar-trade return from a long position in the Foreign ∞-maturity bond funded by a short position in the Home ∞-maturity bond for one period—i.e., the infinite-maturity limit of equation (3), which combines currency and local-currency bond returns. The second pertains to the term structure of convenience yields:

[10] Relative to a model without convenience (as in Alvarez and Jermann, 2005; Lustig et al., 2019) the additional convenience-yield term indicates that, by taking a long position in equities, the Home investor not only forgoes the pecuniary return on the safe one-period bond R_t but also the convenience yield on this bond $\theta_t^{H,H(1)}$. However, quantitatively, we find the effects of convenience yields on measures of risk are very small relative to the equity premium, as illustrated in Figure 4b.

Lemma 3 (Holding-Period Convenience Yields) *Given $M_{t,t+1}$, $M^*_{t,t+1}$, convenience yields satisfy:*

$$\mathbb{E}_t\big[\theta^{F,H(k)}_{t,t+1} - \theta^{H,H(k)}_{t,t+1}\big] = \big(\theta^{F,H(1)}_t - \theta^{H,H(1)}_t\big) + \Phi^{H(k)}_t, \tag{21}$$

$$\mathbb{E}_t\big[\theta^{F,F(k)}_{t,t+1} - \theta^{H,F(k)}_{t,t+1}\big] = \big(\theta^{F,F(1)}_t - \theta^{H,F(1)}_t\big) + \Phi^{F(k)}_t, \tag{22}$$

where, $\forall t,\ \forall k \geq 1$,

$$\Phi^{H(k)}_t \equiv \mathcal{L}_t\big(M_{t,t+k}\big) - \mathcal{L}_t\big(M_{t,t+k}\, e^{-\eta_{t+1}-(\theta^{F,H(k-1)}_{t+1}-\theta^{H,H(k-1)}_{t+1})}\big) + \mathcal{L}_t\big(e^{-\eta_{t+1}}\big) + \mathcal{C}_t\big(M_{t,t+1},\, e^{-\eta_{t+1}}\big),$$

$$\Phi^{F(k)}_t \equiv \mathcal{L}_t\big(M^*_{t,t+k}\, e^{\eta_{t+1}+(\theta^{F,F(k-1)}_{t+1}-\theta^{H,F(k-1)}_{t+1})}\big) - \mathcal{L}_t\big(M^*_{t,t+k}\big) - \mathcal{L}_t\big(e^{\eta_{t+1}}\big) - \mathcal{C}_t\big(M^*_{t,t+1},\, e^{+\eta_{t+1}}\big).$$

Proof: See Appendix A.5. □

In equation (21), the left-hand terms capture the difference in the holding-period convenience yield earned by a Foreign investor purchasing a Home bond of maturity k and unwinding this position one period later, relative to the convenience yield earned by a Home investor purchasing the same bond. Equation (22) analogously captures the holding-period convenience yield earned by a Foreign investor, long in a Foreign k-maturity bond, net the convenience yield earned by a Home investor purchasing the same bond. The terms $\{\Phi^{H(k)}_t, \Phi^{F(k)}_t\}$ are 'convenience term premia', arising to compensate for the riskiness of convenience yields when international markets are incomplete. In the limit of complete international spanning ($\mathcal{L}_t(e^{\eta_{t+1}}) \to 0$), convenience term premia collapse to short-term compensation for the riskiness of convenience yields.[11]

The next proposition details our main theoretical result:

Proposition 2 (SDF Volatility, FX Risk and Convenience Yields at Long Maturities) *Given $M_{t,t+1}$, $M^*_{t,t+1}$, one-period currency portfolio returns from long-term bonds are given by:*

$$\mathbb{E}_t[rx^{CT,(\infty)}_{t+1}] = \underbrace{\mathcal{L}_t(M^{\mathbb{P}}_{t,t+1}) - \mathcal{L}_t(M^{\mathbb{P}*}_{t,t+1})}_{\text{Permanent Risk Differential}} + \underbrace{\mathbb{E}_t[\theta^{F,H(\infty)}_{t,t+1} - \theta^{F,F(\infty)}_{t,t+1}]}_{\text{Long-Maturity Holding-Period Convenience}} - \underbrace{\Omega^{(\infty)}_t}_{\text{IM term}}, \tag{23}$$

[11] Specifically, in the complete-spanning limit:

$$\Phi^{H(k)}_t \to -\mathcal{L}_t\big(e^{-\theta^{F,H(k-1)}_{t+1}+\theta^{H,H(k-1)}_{t+1}}\big) - \mathcal{C}_t\big(M_{t,t+1},\, e^{-\theta^{F,H(k-1)}_{t+1}+\theta^{H,H(k-1)}_{t+1}}\big),$$
$$\Phi^{F(k)}_t \to \mathcal{L}_t\big(e^{+\theta^{F,F(k-1)}_{t+1}-\theta^{H,F(k-1)}_{t+1}}\big) + \mathcal{C}_t\big(M^*_{t,t+1},\, e^{+\theta^{F,F(k-1)}_{t+1}-\theta^{H,F(k-1)}_{t+1}}\big), \forall k, t.$$

We provide analytical conditions for these expressions in Section 3.3.

where $\Omega_t^{(\infty)} = \Phi_t^{H(\infty)} - \mathcal{L}_t(e^{-\eta_{t+1}}) - \mathcal{C}_t(M_{t,t+1},\, e^{-\eta_{t+1}})$.

Proof: See Appendix A.6. □

Bringing equation (1) to bear on long maturities, the proposition above implies that any cross-country asymmetry in permanent risk must be absorbed by some combination of three margins: pecuniary dollar-trade returns, long-maturity holding-period convenience yields or risk premia $\Omega_t^{(\infty)}$. In the data, pecuniary dollar-trade returns on long-maturity bonds are, on average, close to zero, so in equilibrium, convenience yields and risk premia must adjust. Abstracting (for now) from $\Omega_t^{(\infty)}$, if $\mathbb{E}_t[rx_{t+1}^{CT,(\infty)}] \approx 0$, equation (23) predicts that any movement in relative permanent risk is offset one-for-one by the expected holding-period convenience differential. As an example, if Foreign permanent risk exceeds U.S. permanent risk such that $\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}*}) > \mathcal{L}_t(M_{t,t+1}^{\mathbb{P}})$, then, $\mathbb{E}_t[\theta_{t,t+1}^{F,H(\infty)} - \theta_{t,t+1}^{F,F(\infty)}]$– the convenience Foreign investors expect from holding a long-maturity U.S. Treasury for one period rather than a Foreign sovereign– must rise.

3.3 A Tractable Specification with Incomplete Spanning

To gain further insight and guide our empirical work, we discipline $\Omega_t^{(\infty)}$ using (standard) restrictions on SDF and convenience yield processes. We restrict attention to affine SDFs with stochastic-volatility loadings of the square-root form (Cox et al., 1985; Sun, 1992). For ease of exposition, we denote logarithms by lower case (i.e., $m_{t,t+1}^{(*)} = \ln M_{t,t+1}^{(*)}$) and focus on the limit $m_{t,t+1} \to m_{t,t+1}^{\mathbb{P}}$. Specifically, consider the following Home and Foreign SDFs:

$$-\, m_{t,t+1} = \tfrac{1}{2}\gamma^{\mathbb{P}} z_t^{\mathbb{P}} + \sqrt{\gamma^{\mathbb{P}} z_t^{\mathbb{P}}}\; u_{t+1}^{\mathbb{P}}, \qquad -m_{t,t+1}^{*} = \tfrac{1}{2}\gamma^{\mathbb{P}*} z_t^{\mathbb{P}*} + \sqrt{\gamma^{\mathbb{P}*} z_t^{\mathbb{P}*}}\; u_{t+1}^{\mathbb{P}*}, \tag{24}$$

where the innovations $u_{t+1}^{j} \sim \mathcal{N}(0,1)$ are mutually independent across $j \in \{\mathbb{P}, \mathbb{P}^*\}$. One-period convenience yields are driven by *permanent* risk in the two economies:

$$\theta_t^{i,j(1)} \;=\; \alpha^{i,j} \;+\; \pi_{\mathbb{P}}^{i,j}\, z_t^{\mathbb{P}} \;+\; \pi_{\mathbb{P}*}^{i,j} z_t^{\mathbb{P}*}, \tag{25}$$

We define $\delta^j \equiv (\pi_j^{F,H} - \pi_j^{H,H}) - (\pi_j^{F,F} - \pi_j^{H,F})$ and assume for ease of exposition $\delta^j = 0$.[12] We discuss the $\delta \neq 0$ case in Appendix A.8, and we discuss generalizations for both the SDF

[12]Equivalently, $\delta^j = 0$ requires that a Home and a Foreign investor agree on the relative convenience delivered by a U.S. bond. The condition is satisfied automatically under complete spanning.

(24) and the convenience yield (25) processes at the end of the section. The underlying factors $\{z_t^{\mathbb{P}}, z_t^{\mathbb{P}*}\}$ follow independent processes:

$$z_{t+1}^j \;=\; (1-\phi^j)\,\bar{z}^j \;+\; \phi^j\, z_t^j \;-\; \sigma^j \sqrt{z_t^j}\; u_{t+1}^j, \qquad j \in \{\mathbb{P}, \mathbb{P}^*\}, \tag{26}$$

with $\phi^j \in (0,1)$, $\bar{z}^j > 0$, truncated at zero to maintain admissibility, and $\sigma^j > 0$. Direct calculation implies $\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}}) = (\gamma^{\mathbb{P}}/2)\, z_t^{\mathbb{P}}$ and analogously for Foreign risk.

Proposition 2′ (Equilibrium with Incomplete Markets). *Suppose $M_{t,t+1}$, $M_{t,t+1}^*$ and $\theta_t^{i,j(1)}$, for $i,j \in \{H,F\}$, satisfy (24)-(26), one-period dollar-trade returns from long-term bonds are given by:*

$$\mathbb{E}_t[rx_{t+1}^{CT,(\infty)}] = \tilde{\xi}\,\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}}) - \tilde{\xi}^*\,\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}*}) + \mathbb{E}_t[\theta_{t,t+1}^{F,H(\infty)} - \theta_{t,t+1}^{F,F(\infty)}] \tag{27}$$

where $\tilde{\xi} \equiv \xi^{\mathbb{P}} - c^{\mathbb{P}}, \tilde{\xi}^* \equiv \xi^{\mathbb{P}^*} - c^{\mathbb{P}^*}$. The coefficients $\xi^{\mathbb{P}}, \xi^{\mathbb{P}*} \in [0,1]$ are ratios of spanned-to-total variances of SDFs and $\{c^{\mathbb{P}}, c^{\mathbb{P}*}\}$ govern $\Phi_t^{H(\infty)} = c^{\mathbb{P}}\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}}) - c^{\mathbb{P}*}\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}*})$.

Proof: See Appendix A.8. □

When international financial markets are incomplete, non-traded risk can lead to different coefficients on U.S. and Foreign risk, even abstracting from convenience yield term premia captured within $\Phi_t^{H(\infty)}$. We explain each element of (27) in turn. First, the loadings $\xi^{\mathbb{P}(*)} \equiv \kappa^{\mathbb{P}(*)}/\gamma^{\mathbb{P}(*)}$ capture the ratio of spanned-to-total variance of SDFs. Under complete spanning ($\xi^{\mathbb{P}} = \xi^{\mathbb{P}*} = 1$) this channel is shut down: U.S. and Foreign permanent risk enter the long-bond dollar trade with symmetric weights, and any positive average premium comes entirely from cross-country dispersion in $\mathcal{L}_t(M^{\mathbb{P}(*)})$ itself—unless the convenience term premia themselves load asymmetrically ($c^{\mathbb{P}} \neq c^{\mathbb{P}*}$). In turn, $c^{\mathbb{P}(*)}$ capture how convenience term premia themselves propagate risk across maturities. Taken together, asymmetries in $\xi^{\mathbb{P}(*)}$ and $c^{\mathbb{P}(*)}$, can, for example, capture that the dollar's institutional role may make Foreign demand for U.S. Treasuries more (or less) risk-elastic than the Foreign demand for own-currency sovereigns, amplifying (dampening) the response of the U.S. basis to U.S. risk.[13] Moreover, while the theory does not sign $\xi^{\mathbb{P}} - c^{\mathbb{P}}$, stan-

[13] It is interesting to contrast our results to Lustig and Verdelhan (2019), who emphasize that trade in additional risky assets internationally restricts the dynamics of η_{t+1}, enough to restore complete markets. Here, this does not happen because each asset maturity carries a distinct wedge governed by $\Phi^{H(k)}, \Phi^{F(k)}$.

dard calibrations suggest it is positive, since otherwise convenience-yield term premia would exceed exchange-rate premia in size. While we do not separately identify $\xi^{\mathbb{P}}$ and $c^{\mathbb{P}}$, the data pin down $\tilde{\xi} > 0$ and $\tilde{\xi}^* > 0$ (Table 2, column (2)).

Generalizing SDF and Convenience-Yield Processes. Several model variants generalize our results and motivate additional controls in our empirical specifications. We briefly summarize two of them, deferring details to Appendix A.9. First, allowing SDFs to be driven also by transitory innovations implies an additional term in (27)—but we do not find empirical support for this. Second, we can allow convenience yields to depend also on orthogonal factors $z_t^{\theta(*)}$ (e.g., fiscal considerations, volatility indices or intermediary leverage), introducing an additional term, $-c^{\theta} z_t^{\theta} + c^{\theta *} z_t^{\theta *}$ and motivating additional controls in our regressions.

4 Data and Measurement

We now describe the data used to evaluate our propositions. This requires measures of currency returns, risk and convenience (both within- and across-countries). We focus on advanced-economy currencies where financial-market data is sufficiently rich to capture these quantities: Australia (AUD), Canada (CAD), Switzerland (CHF), euro area/Germany (EUR), Japan (JPY), UK (GBP) and US (USD). For our regression-based analysis, we henceforth use a monthly panel over the period 2000:01-2021:03. We use equation (3) and (15) to construct measures of currency returns, $rx_{t+1}^{CT,(k)}$ and rx_{t+1}^{FX}, respectively, following a common approach in the literature by using *ex post* exchange rates and yields to proxy for *ex ante* expectations for our benchmark measure. Underlying exchange-rate data is from *Datastream*, with the USD as the base currency. We supplement this with *Consensus Economics* 6-month exchange rate forecasts for robustness. We then use data on the term structure of interest rates from government bond markets in each AE. Yield curves are obtained from a combination of sources, including national central banks, Anderson and Sleath (2001), Gürkaynak et al. (2007), and Wright (2011). Nominal 6-month zero-coupon bond yields provide our measure of short-term 'safe' interest rates. Like Lustig et al. (2019) and others, 10-year nominal zero-coupon bond yields serve as our proxy for the infinite-maturity bond yield, which amounts to assuming that the yield curve is sufficiently flat beyond the 10-year horizon. We construct bond term premia for each AE jurisdiction using the Adrian et al. (2013) decomposition of bond yields for our baseline measure.[14] Data on dividend-price ratios and equity-price indices for each AE currency area are from *Global Financial Data*, while data for buybacks are from *Bloomberg* (see Dison and Rattan, 2017).[15]

4.1 Measuring Risk

We measure *ex ante* equity risk premia, $\log \mathbb{E}_t[R^{eq}_{t,t+1}/R_t]$, using equation (2), and we construct measures of expected 6-month-ahead dividend growth as the fitted value from a regression of future 6-month dividend growth on the dividend yield, lagged dividend growth and *Consensus Economics*' next-calendar-year GDP growth survey expectations, as detailed in Appendix B.2.

Assuming (19) holds with equality, our proxy for total SDF volatility, $TotRisk_t$ is:[16]

$$TotRisk_t := \log \mathbb{E}_t\left[\frac{R^g_{t,t+1}}{R_t}\right] - \mathcal{L}_t\left[\frac{R^g_{t,t+1}}{R_t}\right], \tag{28}$$

where the two terms capture the expected log equity premium. The first term is the log *expected* return on the riskiest portfolio in the economy– constructed from our valuation-based measure, equation (2). The second term ensures that our proxy for total risk is corrected for the expected volatility of returns, $\mathcal{L}_t\left(R^g_{t,t+1}/R_t\right)$. We construct a proxy for conditional volatility using rolling daily averages of past data and compare our measure to a traded volatility index (VIX_t) for the U.S. in Appendix B.2. Because the dependent variable in our regressions is itself a convenience yield, we omit the within-country convenience term from our risk measure—it accounts for only a tiny fraction of measured risk (see Figure 4b)—so that convenience does not enter on both sides of the regression.

In turn, assuming (20) holds with equality, our proxy for permanent risk, $PermRisk_t$, is:

$$PermRisk_t := \log \mathbb{E}_t\left[\frac{R^g_{t,t+1}}{R_t}\right] - \mathcal{L}_t\left[\frac{R^g_{t,t+1}}{R_t}\right] - \mathbb{E}_t[rx^{(10Y)}_{t+1}] \tag{29}$$

[14] In robustness analysis, we use realized excess returns on bonds as an alternative measure.

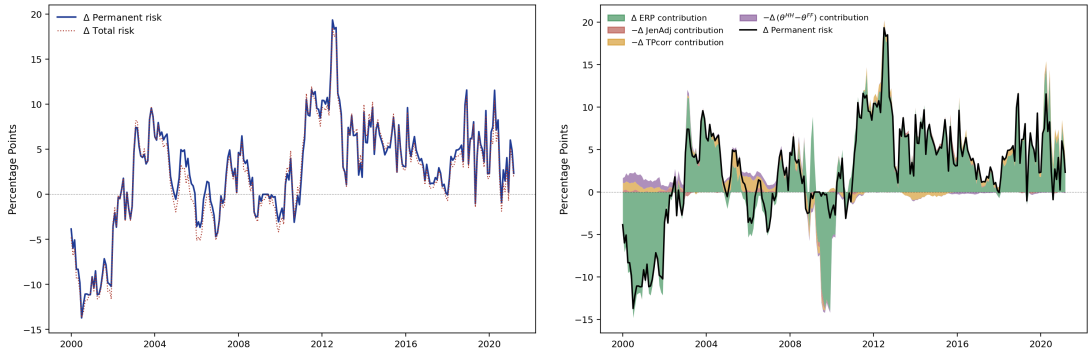
[15] Specifically, we use the S&P-500 for the US, EuroStoxx-50 for the EA, FTSE-100 for the UK, TOPIX for Japan, S&P/ASX-200 for Australia, S&P/TSX-300 for Canada and SMI for Switzerland.

[16] Our empirical results are robust to using a leveraged equity portfolio, constructed as the growth-optimal portfolio (Bansal and Lehmann, 1997), designed to maximize the RHS of (19). Theoretically, this equality arises naturally within the one-factor affine framework specified in Section 3.3, or when financial markets are dynamically complete within countries (see Appendix A.10).

Figure 4: Relative U.S. Permanent and Total Risk

(a) U.S. vs AE Permanent and Total Risk

(b) Decomposition of Δ Permanent Risk



Note. Panel (a): U.S. minus equally-weighted AE permanent- and total-risk differentials. Panel (b): decomposition of the permanent-risk differential into ΔERP, $-\Delta$JenAdj, $-\Delta$TPcorr, and the within-country convenience wedge; the thick black line is the exact differential, which departs from the sum of components when the truncation $\max\{0, \cdot\}$ binds. Sample: 2000:01–2021:03.

The first two terms follow from equation (28). The third term accounts for domestic long-maturity bond premia, for which we use the Adrian et al. (2013) yield term premium for each advanced economy as our baseline empirical proxy. We construct total and permanent risk for each advanced economy, truncating to ensure positivity. We use U.S. risk for Home and the cross-sectional average of other advanced economies for Foreign, to construct total ($TotRisk_t^*$) and permanent ($PermRisk_t^*$) proxies. We define cross-country differences by $\mathcal{D}$, such that $\mathcal{D}TotRisk_t := TotRisk_t - TotRisk_t^*$ and $\mathcal{D}PermRisk_t := PermRisk_t - PermRisk_t^*$.[17]

Figure 4 presents our proxies for *relative* U.S. permanent and total risk (Panel 4a), as per Lemma 2, and the decomposition of the former into its contributing parts (Panel 4b). Similar to expected relative equity premia, relative U.S. total risk has generally increased over the past two decades, albeit with pronounced troughs during the aftermath of the dot-com bubble and the GFC, and attributable almost entirely to a rise in relative U.S. permanent risk. We discuss country-by-country risk measures and their drivers further in Appendix B.2.

4.2 Measuring Convenience Yields

To measure the excess convenience a Foreign investor earns on a U.S. Treasury ($\theta_t^{F,H(k)} - \theta_t^{F,F(k)}$), we use data on the Treasury basis from Du et al. (2018a). We construct short-maturity cross-

[17]The measure of U.S. permanent risk we use is positive for over 90% of our sample. It is well known that it can yield negative values when the ERP is small or negative, see Alvarez and Jermann (2005), which in our sample occur almost exclusively around the GFC. In line with the literature, we ensure that country series are weakly positive setting negative entries to zero, visible in Figure 4b during the GFC, see also Figure B.5b.

country convenience using 6-month CIP deviations, $CIP_t^{(6M)}$,[18] and the 10-year tenor $CIP_t^{(10Y)}$ for long-maturity cross-country convenience. However, CIP deviations are still a step removed from convenience yields. The literature has recognized that convenience may be related to either the 'dollarness' of U.S. Treasuries or the Treasuries themselves, or both, so may not exactly correspond to the Treasury basis.

Focusing on short-maturities, Jiang et al. (2021a) propose a method to estimate the relative importance of the two that, in effect, scales-up observable CIP deviations to construct a proxy for the cross-country convenience yields inherent in holding US Treasuries. Consider foreign investors' convenience on synthetic U.S. Treasuries (i.e., Foreign government bonds swapped into dollars using a forward contract). This position earns the convenience of a Foreign bond, $\theta_t^{F,F(k)}$, plus some maturity-specific fraction χ_k^* of relative U.S. Treasury convenience, $\theta_t^{F,H(k)} - \theta_t^{F,F(k)}$, resulting in the following proportionality with CIP deviations:[19] $\theta_t^{F,H(k)} - \theta_t^{F,F(k)} = \frac{1}{1-\chi_k^*} CIP_t^{(k)}$. Jiang et al. (2021a) identify χ_k^* under the assumptions that convenience yields are stationary and independent from risk premia. While we verify these assumptions and reproduce their findings for short-maturity U.S. Treasuries (see Appendix B.3), both assumptions are invalid for long-maturity Treasuries. Instead, in our analysis we proxy convenience yields directly by CIP deviations since scaling does not change the relevant statistical properties of the series as long as χ_k^* is constant—so our estimated coefficients implicitly capture the scaling factor.

In addition, Proposition 2 clarifies that *holding-period* convenience—i.e., the *expected* convenience earned on a long-maturity bond position that is unwound a period later—is the relevant quantity in the relationship between currency returns, risk and convenience with long-maturity bonds. Consider a decomposition of convenience over a k-maturity portfolio's lifetime into a

[18] Our 6-month CIP deviation is derived from the Du et al. (2018a) 3-month and 1-year CIP deviation data by linear interpolation, such that: $CIP_t^{(6M)} = \frac{2}{3} CIP_t^{(3M)} + \frac{1}{3} CIP_t^{(1Y)}$. We use the 6-month convenience yield to match the maturity from our zero-coupon bond data.

[19] The Euler equation for the synthetic position is:

$$\mathbb{E}_t \left[M_{t,t+k}^* \frac{\mathcal{E}_t}{\mathcal{E}_{t+k}} \left(\frac{F_t^{(k)}}{\mathcal{E}_t} R_t^{(k)*} \right) \right] = e^{-\theta_t^{F,F(k)} - \chi_k^* (\theta_t^{F,H(k)} - \theta_t^{F,F(k)})}$$

where the term in round brackets on the left-hand side is the synthetic k-period U.S. Treasury pecuniary return. Comparing this with equations (4) and (8), results in the linear, maturity-specific, relationship between the cross-country convenience yield and the CIP deviations. If $\chi_k^* = 1$, the Foreign investor values the synthetic Treasury exactly the same as a U.S.-issued Treasury, suggesting convenience arises from currency. If $\chi_k^* < 1$, then there is intrinsic convenience in government bonds. As long as $\chi_k^* \in (0, 1)$, small CIP deviations correspond to large convenience yields. We adopt a similar strategy for $\theta_t^{H,H}$ and $\theta_t^{F,F}$ using swaps, detailed in Appendix B.4.

flow and a continuation value:

$$\theta_t^{F,H(k)} - \theta_t^{F,F(k)} = c_k + \omega^{(k)} \left(\theta_t^{F,H(k)} - \theta_t^{F,F(k)}\right) + \mathbb{E}_t[\theta_{t+1}^{F,H(k-1)} - \theta_{t+1}^{F,F(k-1)}]$$

for some constant c_k, for all $k > 1$. Investors receive a share $\omega^{(k)} \in (0,1)$ of their portfolio's convenience-to-maturity, $(\theta_t^{F,H(k)} - \theta_t^{F,F(k)})$ and expect to forgo the convenience on a $k-1$-maturity bond, $(\theta_{t+1}^{F,H(k-1)} - \theta_{t+1}^{F,F(k-1)})$, when selling the bond tomorrow. Taking the limit $k \to \infty$, the holding-period convenience yield is given by $\mathbb{E}_t[\theta_{t,t+1}^{F,H(\infty)} - \theta_{t,t+1}^{F,F(\infty)}] = c_\infty + \omega^{(\infty)}(\theta_t^{F,H(\infty)} - \theta_t^{F,F(\infty)})$. In the baseline (symmetric) affine framework specified in Section 3.3, $\omega^{(\infty)} = 1 - \phi$, see Appendix A.11 for a derivation. As such, proxying long-maturity convenience with the 10-year CIP deviation, our long-maturity regression coefficients implicitly capture both the constant weight $\omega^{(k)}$ and $\frac{1}{1-\chi_k^*}$. Even within the affine framework, there is no guarantee that $\omega^{(k)}/(1-\chi^{*k})$ is common across country pairs, or time-invariant beyond the symmetric limit. As such, we consider country-specific variation and sub-sample stability as robustness.

5 Estimating Equilibrium Relationships

To what extent is the decline in long-maturity U.S. convenience yields and the increase in U.S. risk over the past decades connected in the data? We estimate linear relationships between CIP deviations and relative permanent risk to quantify the importance of risk for the long-run evolution of convenience yields. For our baseline, we compare the U.S. to average values for the other advanced economies, but we investigate the panel as robustness.

5.1 Long-Maturity Convenience and Permanent Risk in the Long Run

Our focus is on the relationship between long-maturity dollar-trade returns ($rx_{t+1}^{CT(10Y)}$), relative permanent risk ($\mathcal{D}PermRisk_t$), and the convenience yield foreigners earn when holding U.S. long-maturity bonds for a single period (proxied with $CIP_t^{(10Y)}$). Condition (23) requires the nonstationary component of risk to be absorbed by *some* combination of measured convenience and the deviation term $\Omega_t^{(\infty)}$, but is silent on the split. Our core result is that there is a long-run relationship between these three variables: specifically, a negative association between relative

Table 1: Stationarity and Cointegration Tests

Variable	ADF Test Statistic Without Trend	With Trend
Panel A: Long-Maturity Variables		
$CIP_t^{(10Y)}$	-2.340	-3.073
$\mathcal{D}PermRisk_t$	-2.226	-2.137
$rx_{t+1}^{CT,(10Y)}$	-4.222^{***}	-4.242^{***}
Panel B: Short-Maturity Variables		
$CIP_t^{(6M)}$	-3.442^{**}	-3.444^{**}
$\mathcal{D}TotRisk_t$	-2.196	-2.059
rx_{t+1}^{FX}	-4.242^{***}	-4.362^{***}

Notes: Augmented Dickey–Fuller statistics with 6 lagged differences (using Said–Dickey rule for $T = 255$). Sample: 2000:01–2021:03. ***, **, *: rejection at 1%, 5%, 10%.

permanent risk and long-maturity CIP deviations.[20]

To show this, we first run stationarity tests on our long- and short-maturity variables, applying augmented Dickey–Fuller (ADF) tests (Dickey and Fuller, 1979)—with and without a deterministic time trend and with six lagged monthly changes of each variable to absorb residual autocorrelation. Table 1, Panel A reports the results for the long-maturity variables. The unit-root null is strongly rejected for the realized 10-year dollar-trade return at the 1% level, both with and without a deterministic trend. By contrast, the unit root cannot be rejected for relative permanent risk at even the 10% level, with or without a deterministic trend. We likewise fail to reject the unit-root null for CIP deviations in both specifications. Results differ for the short-maturity variables detailed in Panel B, where the unit-root null is rejected at the 1% level for the 6-month dollar-trade return and at the 5% level for 6-month CIP deviations.

Given this evidence, we characterize the relationship between long-maturity convenience, relative permanent risk and long-maturity dollar-trade returns by estimating variants of the following error-correction model, derived from an autoregressive distributed-lag model implied by Proposition 2′:

$$\begin{aligned}\Delta CIP_t^{(10Y)} =& \beta_0 + \beta_1 \Delta \mathcal{D}PermRisk_t + \beta_2 \Delta rx_{t+1}^{CT(10Y)} \\ &+ \gamma \left[CIP_{t-1}^{(10Y)} - \alpha_0 - \alpha_1 \mathcal{D}PermRisk_{t-1} - \alpha_2 rx_t^{CT(10Y)} - \tau(t-1) \right] + \varepsilon_t, \quad (30)\end{aligned}$$

[20]Our results, however, do not hinge on cointegration—we discuss the validity of our results under mixed integration orders or when both are assumed (trend) stationary.

where $CIP_t^{(10Y)}$ (and its changes) are measured in basis points, while other variables are measured in percentage points. The first-difference terms, pre-multiplied by β_i for $i = 1, 2$, capture the contemporaneous response of long-maturity convenience to changes in relative permanent risk and long-maturity dollar-trade returns—i.e., short-run adjustment. The term pre-multiplied by α_i for $i = 1, 2$ captures the cointegrating relationship implied by the data. The coefficients α_i for $i = 1, 2$ (discussed further below) capture the long-run multipliers between long-maturity convenience, relative-permanent risk and dollar returns, respectively. In turn, the coefficient γ reflects the adjustment in long-maturity convenience to deviations from the long-run relationship.

We estimate regression (30) in two steps, following Engle and Granger (1987). While static OLS delivers super-consistent long-run coefficients α_i, their t-ratios are not asymptotically normal because of second-order terms and endogeneity (Stock, 1987; Phillips and Hansen, 1990). We therefore estimate the long-run coefficients by *dynamic OLS* (DOLS) (Saikkonen, 1991; Stock and Watson, 1993):[21]

$$CIP_t^{(10Y)} = \alpha_0 + \alpha_1\, \mathcal{D}PermRisk_t + \alpha_2\, rx_{t+1}^{CT(10Y)} + \tau\, t + \sum_{i=-4}^{4} b_i\, \Delta \mathcal{D}PermRisk_{t-i} + \varepsilon_t, \quad (31)$$

The long-run coefficients (α_i) are reported in Panel A, Table 2 and the second-stage coefficients are reported in Panel B.

Our baseline specification (Column (1)) includes a deterministic trend and no additional controls. It highlights a significant and negatively-signed cointegrating relationship between long-maturity convenience yields and relative permanent risk, with long-maturity dollar-trade returns playing a secondary role. The coefficients in Panel A reveal that a 1p.p. increase in U.S. permanent risk relative to the remaining advanced economies is associated with approximately a 1.3b.p. decline in U.S. long-maturity CIP deviations over the long run. Column (2) introduces U.S. and Foreign risk separately. We find both are significant with opposite signs, consistent with $\tilde{\xi}, \tilde{\xi}^* > 0$. Point estimates suggest that U.S. permanent risk moves convenience yields by more than Foreign permanent risk, though we cannot statistically reject that the two loadings are equal in absolute value. Excluding the deterministic trend (Column (3)), the association is

[21]The leads and lags of the first-differenced regressor eliminates second-order bias and the endogeneity simultaneously; Newey and West (1987) HAC t-statistics therefore have standard limiting distributions. DOLS is also used in Lettau and Ludvigson (2001) and Gourinchas and Rey (2007), amongst others.

Table 2: Cointegration and Error-Correction Amongst Long-Maturity Variables

	(1) Baseline	(2) Baseline + Split	(3) No Controls	(4) Debt, No Trend	(5) Debt + Leverage
Panel A: Long-Run Adjustment					
$\mathcal{D}PermRisk_t$	-1.330^{***}		-2.681^{***}	-1.259^{***}	-1.305^{***}
	(0.270)		(0.404)	(0.398)	(0.356)
$PermRisk_t$		-1.322^{***}			
		(0.292)			
$PermRisk_t^*$		0.850^{**}			
		(0.397)			
$rx_{t+1}^{CT(10Y)}$	0.203^{**}	0.202^{**}	0.299^{**}	0.212^{**}	0.077
	(0.098)	(0.084)	(0.150)	(0.100)	(0.109)
$\text{B/GDP}_t^{US} - \overline{\text{B/GDP}}_t^*$				-1.338^{***}	-1.609^{***}
				(0.182)	(0.211)
Leverage (1/cap. ratio)					0.790^{***}
					(0.264)
Deterministic Trend	-0.164^{***}	-0.176^{***}			
	(0.017)	(0.016)			
R^2 of Long-Run	0.780	0.789	0.463	0.712	0.741
Panel B: Short-Run Adjustment					
$\Delta\mathcal{D}PermRisk_t$	-0.392^{***}		-0.368^{**}	-0.356^{**}	-0.383^{***}
	(0.143)		(0.149)	(0.143)	(0.146)
$\Delta PermRisk_t$		-0.430^{**}			
		(0.173)			
$\Delta PermRisk_t^*$		0.233			
		(0.335)			
$\Delta rx_{t+1}^{CT(10Y)}$	0.115	0.109	0.108	0.114	0.027
	(0.075)	(0.073)	(0.082)	(0.077)	(0.093)
Δ Debt/GDP differential				-0.069	-0.783
				(0.974)	(0.866)
Δ Leverage					0.802^{**}
					(0.336)
Diseq. Adjustment $\hat{\gamma}$	-0.235^{***}	-0.244^{***}	-0.099^{***}	-0.179^{***}	-0.193^{***}
	(0.043)	(0.046)	(0.023)	(0.039)	(0.045)
Adj. R^2 of ECM	0.119	0.118	0.054	0.088	0.127
Engle–Granger Test Stat.	-5.47^{***}	-5.46^{***}	-4.14^{***}	-4.82^{***}	-4.98^{***}
Deterministic Trend	Yes	Yes	No	No	No
# Observations (Panel A / B)	246 / 254	246 / 254	246 / 254	246 / 254	246 / 254

Notes: Panel A: long-run regression of 10-year CIP deviations (b.p.) on relative permanent risk (p.p.), dollar-trade returns, and the indicated controls, estimated by dynamic OLS with four leads and lags of each first-differenced $\mathcal{I}(1)$ regressor; the reported R^2 refers to the static levels regression. Column (2) splits relative risk into its U.S. and AE components; column (5) adds the inverse He et al. (2017) capital ratio. Panel B: error-correction regression using the disequilibrium term implied by the DOLS coefficients. The Engle–Granger ADF tests the static first-stage residual; Newey–West (6 lags) standard errors in parentheses, while stars on the Engle–Granger statistic use MacKinnon (2010) finite-sample critical values with N equal to the number of $\mathcal{I}(1)$ variables in each column's long-run regression. ***, **, *: 1%, 5%, 10%. Sample: 2000:01–2021:03.

stronger—up to almost 2.7b.p.[22]

Column (1) of Panel B also highlights that, all else equal, higher permanent risk contributes to lower relative convenience not only in the long run, but also contemporaneously—a result which is robust in all specifications. The disequilibrium-adjustment coefficient $\hat{\gamma} = -0.235$, significant at 1%, implies that roughly a quarter of any deviation from the long-run relationship is corrected each month.

The deterministic trend is included in our baseline specification to account for competing secular factors that could explain the decline in long-maturity convenience yields. To dig deeper into the economic drivers, column (4) introduces the difference in U.S. and Foreign debt-to-GDP ratios (see e.g. Krishnamurthy and Vissing-Jorgensen, 2012; Jiang et al., 2024). The coefficient on this variable is both significant and intuitive: a 1p.p. increase in the relative U.S. debt-to-GDP is associated with a 1.3b.p. decrease in the U.S. 10-year convenience yield, controlling for relative permanent risk and dollar returns. Nevertheless, the estimated coefficients on the level and change in relative permanent risk are largely unchanged—signaling the role of permanent risk *over and above* the scarcity of U.S. debt.

In column (5), we additionally include the intermediary leverage ratio—the inverse of the He et al. (2017) capital ratio—a series that is high when dealer balance sheets are most constrained. As shown, this enters the long-run relationship positively and significantly. Both in the long-run and cyclically, a one unit increase in intermediary leverage is associated with a 0.8b.p. increase in the long-maturity CIP deviation.

Across the specifications, three results are worth stressing. First, although both a relatively higher U.S. debt-to-GDP ratio and a higher capital ratio imply lower long-maturity CIP deviations, the loading on relative permanent risk is very close to the baseline. As such, the risk channel we characterize operates over and above both debt and intermediary frictions. Second, long-maturity dollar-trade returns enter the long-run relationship with the sign predicted by theory, but their adjustment is never sufficient to restore equilibrium alone. Finally, note that the specification in column (5), which does not include the deterministic trend, explains almost the same variation as our baseline (column (1)), suggesting that, jointly, debt and leverage

[22]The coefficients in Table 2 are essentially unchanged if $rx_t^{CT(10Y)}$ is removed from the long-run relationship, leaving only $CIP_{t-1}^{(10Y)}$ and $\mathcal{D}PermRisk_{t-1}$ in the first-stage.

account for almost the entirety of the trend in convenience yields.[23]

5.2 Contribution of Permanent Risk to Long-Maturity Convenience

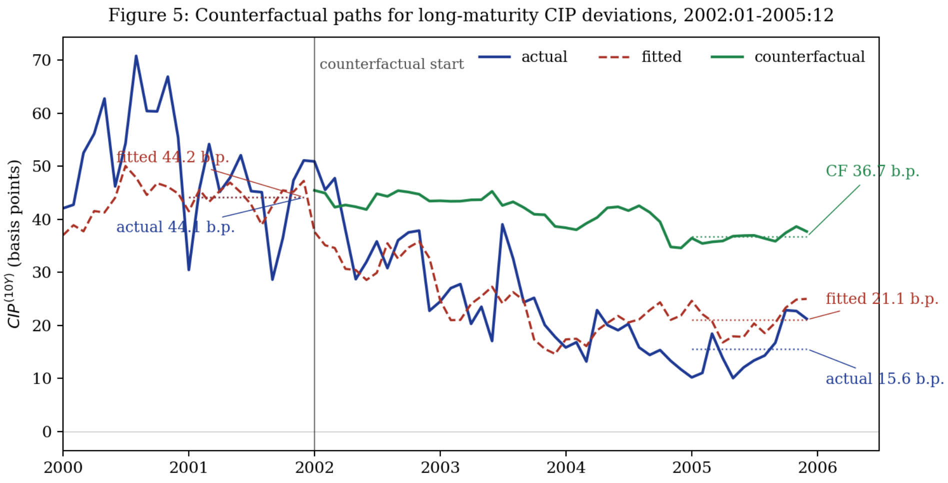
To illustrate the economic significance of our results, we use the estimated coefficients in column (1) to carry out historical counterfactual exercises. Based on the model, we ask: given the path of long-maturity dollar-trade returns and the deterministic trend, how would long-maturity CIP deviations have evolved had U.S. permanent risk not increased over time? Since the fitted and counterfactual paths share every input except risk, the counterfactual gap at each date is $\hat{\alpha}_1\big(\mathcal{D}PermRisk_t - \overline{\mathcal{D}PermRisk}\big)$: the estimated long-run coefficient (Table 2) times the deviation of relative permanent risk from its frozen pre-episode mean.

In our first experiment, we fix relative permanent risk to its average value between 2000:01 to 2001:12, near its dot-com bubble trough. We construct a counterfactual path for long-maturity convenience yields over the four-year period 2002:01 to 2005:12. Figure 5 plots this counterfactual path alongside the actual evolution of 10-year CIP deviations and the fitted path from our empirical model. At the starting point in 2002:01, 10-year convenience yields are 44.1b.p. The model fits the actual data well and, most importantly, by 2006 the model predicts a fall in convenience yields to 21.1b.p., a 23.1b.p. drop, compared to a decline in the actual data of 28.5b.p. (down to 15.6b.p.), over the same period—so the model captures four fifths of the secular decline. In contrast, when abstracting from the increase in relative U.S. permanent risk over the 2002-2006 period, CIP deviations decline by much less. Specifically, the counterfactual path for CIP deviations deviates from the empirical path from the start of the 4-year period, and declines gently by only 7.4b.p., to 36.7b.p. Through the lens of our model, the relative rise in U.S. permanent risk in the years following the dot-com bubble (and preceding the GFC) rationalizes around 68% of the decline in the fitted long-maturity U.S. Treasury convenience yields, and around 55% of the decline observed in the data.

Since this gap between the fitted and counterfactual paths is simply $\hat{\alpha}_1$ times the movement in relative permanent risk, an asymptotic confidence interval follows directly from the standard error in Table 2, column (1): the risk contribution of 15.6b.p. carries a 95% interval of $[8.4, 22.8]$b.p. The corresponding interval for its share of the fitted decline is $[53\%, 75\%]$; this

[23] A saturated specification including the trend alongside debt and leverage raises the R^2 only to 0.79. Within it, dropping relative risk lowers the R^2 by 8p.p., against 5 for the trend and 1 for debt and leverage together. As discussed in robustness, the R^2 for permanent risk in a detrended regression is 24%.

Figure 5: Counterfactual Paths for Long-Maturity CIP Deviation 2002:01-2005:12



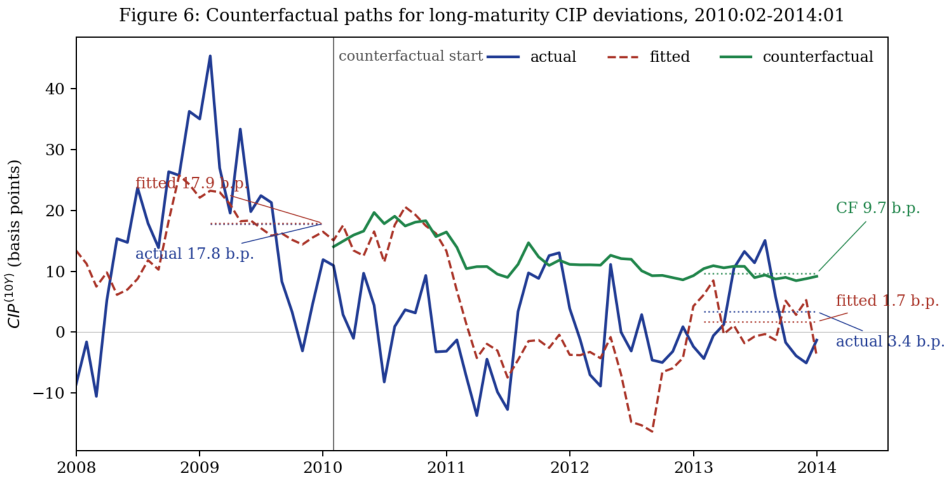
Notes: Actual, fitted, and counterfactual paths for the 10-year CIP deviation, constructed from the long-run relationship (31) with coefficients from Table 2, column (1). The counterfactual fixes relative permanent risk at its 2000:01–2001:12 average over 2002:01–2005:12; all other series follow the data.

interval is computed treating the remaining coefficients as fixed, and accounts for the fact that varying $\hat{\alpha}_1$ moves the fitted decline along with the risk contribution.

In the second experiment, we fix relative permanent risk near its trough around the GFC. Here, we calculate the average value of relative permanent risk from 2009:02 to 2010:01. We then fix relative permanent risk at this level from 2010:02 to 2014:01. Figure 6 compares the actual, fitted and counterfactual paths for 10-year CIP deviations. The model slightly overestimates the 14.5b.p. decline in CIP deviations, with actual end values averaging 3.4b.p. and fitted values averaging 1.7b.p. over the period 2013:02-2014:01. Compared to fitted values, the counterfactual path starts to diverge during 2011, from which point the counterfactual declines at a much slower pace. Over the 2013:02-2014:01 period, the counterfactual path for 10-year CIP deviations is 8.0b.p. above the fitted values relative to the full model. That is, over this period, permanent risk differentials rationalize around 49% of the fitted decline in convenience yields, and again around 55% of the decline observed in the data. As before, we calculate the associated 95% interval as $[4.3, 11.7]$b.p., or $[34\%, 58\%]$ of the fitted decline.

Both experiments lend strong support, at least for long-maturity assets, to the prediction of the class of models which suggest that in equilibrium, risk differentials are compensated by either pecuniary *or* non-pecuniary returns, in line with variants of condition (1). The attribution is

Figure 6: Counterfactual Paths for Long-Maturity CIP Deviation 2010:02-2014:01.



Notes: Actual, fitted, and counterfactual paths for the 10-year CIP deviation, constructed from the long-run relationship (31) with coefficients from Table 2, column (1). The counterfactual fixes relative permanent risk at its 2009:02-2010:01 average over 2010:02 to 2014:01; all other series follow the data.

robust to how the secular component is modeled.[24] In the data, dollar-trade (pecuniary) returns do not move enough to compensate for significant movements in relative permanent risk. As a result, within our estimated cointegrating framework, when perceived U.S. risk increases, reflected in higher expected returns for risky U.S. assets, cross-country convenience yields (i.e., CIP deviations) on long-maturity assets adjust downwards, reflecting a fall in Foreign investors' preferences for U.S. bonds.

5.3 Robustness

Other Risk Measures. In Column (1) of Table 3, we add the VIX, a traded volatility index, as an alternate measure of U.S. and global risk. While our headline result for permanent risk is largely unchanged, the VIX is highly significant for long-run adjustment: a 1p.p increase in the VIX corresponds to a 0.28b.p. increase in long-maturity convenience yields. Foreigners prefer U.S. assets during periods of uncertainty once we have controlled for permanent risk differentials. Indeed, the result is not robust to *replacing* our preferred measures of risk with

[24]Replacing the deterministic trend with debt and leverage leaves the absolute contribution of relative risk essentially unchanged, at 15.3b.p. (against 15.6) in the first episode and 7.8b.p. (against 8.0) in the second. The corresponding shares of the fitted declines are 64% and 40%, against 68% and 49% in the baseline. The contributions are similarly stable if the counterfactual is instead simulated through the full error-correction model (allowing short-run adjustment), landing within 2b.p. of these figures in both episodes.

stationary measures related to the (S)VIX (see, e.g., Martin, 2017).[25] Moreover, Table B.7 in Appendix B.5 shows our results are robust to two alternative formulations of our risk measure: the first supposes buybacks are unexpected, the second considers the growth-optimal portfolio.

Generated Regressors and Inference. A key input into $\mathcal{D}PermRisk_t$—expected payout growth—is itself constructed from country-level forecasting regressions, so our regressor embeds first-stage sampling uncertainty that conventional standard errors ignore (Pagan, 1984). To account for this, we use a moving-block bootstrap as in, e.g., Kremens and Martin (2019). We resample the first-stage estimation sample (267 months) in 24-month blocks, drawn coincidentally across the seven countries to preserve the cross-country correlation of coefficient errors, re-estimate the regression country-by-country, rebuild the risk measures, and re-estimate the long-run DOLS coefficient, repeating this $B = 1000$ times.[26] Accounting for the variance of the long-run coefficient across the first-stage bootstrap raises the standard error on the long-run coefficient from 0.270 to 0.312 resulting in a 95% confidence interval $[-1.94, -0.72]$ but leaving the significance at the 1% level unaffected.

Survey Expectations of Exchange Rates. Column (2) in Table 3 replaces realized dollar-trade returns with expected returns constructed using 6-month-ahead *Consensus Economics* survey forecasts for the exchange rate. The coefficient on survey-based expected long-maturity dollar-trade return is positive and roughly twice the size of that on its realized analogue. However, the role of permanent risk differentials is stable and significant.

Placebo Test. Column (3) adds transitory risk to our baseline specification. Consistent with our theoretical framework, the coefficient on $\mathcal{D}TransRisk_t$ is insignificant and the coefficient on relative permanent risk is essentially identical to the baseline (-1.33 in both)– indicating that permanent risk specifically drives long maturity CIP deviations.

[25]Specifically, Martin (2017) shows that $\sigma_t(M_{t,t+1})\,\text{var}_t^{RN}(R^g_{t+1}) \geq ERP_{t+1} \geq R_t \times SVIX_t^2$, where var_t^{RN} refers to the conditional risk-neutral variance. Our measure relies on the middle argument in the inequality to proxy for the LHS, whereas the $SVIX_t$ appears in the RHS, which could be increasingly biased over time. Marin and Jung (2026) show the bias between the VIX and ERP may have risen over time.

[26]The 24-month block length balances the persistence of the overlapping growth series against the number of available blocks; results are insensitive to block lengths between 12 and 36 months.

Realized Bond Premia. Column (4) of Table 3 replaces the Adrian et al. (2013) model-implied term premium with a 60-month trailing moving average of realized excess bond returns, which isolates the slow-moving component of ex-post returns without imposing an affine (or stationary) model. Relative permanent risk remains correctly signed and significant. The coefficient falls by about 60% reflecting a noisier resulting risk measure than the baseline.

Table 3: Cointegration Amongst Long-Maturity Variables: VIX Control, Survey-FX dollar-Trade Returns, Placebo with Transitory Risk, and Realized Bond Premia.

	(1) Baseline + VIX	(2) Survey FX	(3) Placebo	(4) Realized Bond Premia
Panel A: Long-Run Adjustment				
$\mathcal{D}PermRisk_t$	-1.242^{***}	-1.115^{***}	-1.331^{***}	-0.502^{**}
	(0.249)	(0.225)	(0.280)	(0.235)
$\mathcal{D}TransRisk_t$			2.193	
			(1.592)	
$rx_{t+1}^{CT(10Y)}$	0.143		0.185^{*}	0.219^{**}
	(0.087)		(0.095)	(0.106)
$\mathbb{E}[rx_{t+1}^{CT(10Y)}]$		0.397^{***}		
		(0.114)		
VIX	0.284^{***}			
	(0.078)			
Deterministic Trend	-0.163^{***}	-0.165^{***}	-0.169^{***}	-0.204^{***}
	(0.015)	(0.018)	(0.017)	(0.019)
R^2 of Long-Run	0.802	0.808	0.781	0.741
Deterministic Trend	Yes	Yes	Yes	Yes
# Observations	246	246	246	246

Notes: Long-run (DOLS) regressions of 10-year CIP deviations (b.p.) on relative permanent risk (p.p.), dollar-trade returns, and a deterministic trend, with four leads and lags of $\Delta\mathcal{D}PermRisk_t$. Column (1) adds the VIX; column (3) adds relative transitory risk (both stationary, entering without augmentation); column (2) replaces realized with survey-based expected returns; column (4) rebuilds permanent risk with a sixty-month trailing average of realized excess bond returns in place of the Adrian et al. (2013) term premium. Newey and West (1987) (6 lags) standard errors in parentheses. Sample: 2000:01–2021:03.

Country-Time Variation. We investigate the long-run equilibrium relationship country by country, allowing for distinct coefficients, as well as across three sub-samples (Full, Pre-GFC, Post-GFC).[27] We summarize our main findings here, leaving a detailed exposition of our results to Appendix B.6 and Table B.8. First, over the full sample, all countries except JPY have a

[27] These results relax the restrictions we impose on the mapping between CIP and holding period convenience $\{\chi^{*(k)}, \omega^{(k)}\}$, see Section 3.3 to be country and time-invariant and continue to support our findings.

negative bilateral loading on relative permanent risk, and the coefficient is significant at the 1% level for AUD, CAD and EUR. Second, the pre-GFC results are stronger—the coefficient is negative and significant at the 1% level for every currency except GBP, including CHF and JPY. Post-GFC, CHF and JPY, the two other safe haven currencies, switch to a positive coefficient, yet the pooled panel coefficient remains negative but insignificant. Excluding CHF and JPY, the point estimate is stable across subsamples and significant at 1%, and the corresponding mean-group estimate is significantly negative in every subsample.

Mixed Orders of Integration and Trend-Stationary Alternative. While for cointegration both CIP deviations and relative permanent risk are to be $I(1)$, our estimation is robust to mixed integration orders. Note that if both $CIP_t^{(10Y)}$ and $\mathcal{D}PermRisk_t$ were $I(1)$ but *would not* cointegrate, this could result in a spurious regression, but this case is ruled out because our first-stage residual is $I(0)$. The Engle–Granger residual ADF tests reject the unit-root null at the 1% level in the baseline specification. If, instead, $CIP_t^{(10Y)}$ and $\mathcal{D}PermRisk_t$ were $I(0)$ around a deterministic trend, the level coefficient on $\mathcal{D}PermRisk_t$ would remain identified, and OLS would suffice. As a check, we also estimate the coefficient in a regression of de-trended CIP on de-trended relative permanent risk, which yields -1.10, well within the range spanned by our estimators, and accounts for 24% of the variation in de-trended CIP deviations.

5.4 Short-Maturity Convenience and Risk

We conclude our analysis contrasting our results for long-maturities with those from a simple regression of short-maturity CIP deviations on the corresponding dollar-trade returns and the measure of relative *total* risk, based on Proposition 1. We difference both series to avoid mixed integration orders. Table 4 reveals striking differences between the relationships for short- and long-maturity portfolios. At short maturities, the relationship between relative risk and CIP deviations is not significantly different from zero: convenience on short-maturity bonds appears insulated from total risk. On the other hand, higher short-maturity CIP deviations are weakly associated with a subsequently depreciating USD—assuming interest rate differentials are relatively stable—both using realized returns (Columns (1) & (2)) and survey-based expected returns (Column (4)), in line with Jiang et al. (2021a) and Engel and Wu (2023).

Our short-maturity results are suggestive of within-country market segmentation—i.e., in-

Table 4: Short-Maturity Associations

Dep. Var.: $\Delta CIP_t^{(6M)}$	(1)	(2)	(3)	(4)
Δrx_{t+1}^{FX}	0.392^{*}	0.391^{*}	0.389^{*}	
	(0.232)	(0.232)	(0.230)	
$\Delta \mathbb{E}_t[rx_{t+1}^{FX}]$				0.537^{*}
				(0.310)
$\Delta \mathcal{D}TotRisk_t$		0.488		0.518
		(0.515)		(0.504)
$\Delta TotRisk_t$			0.456	
			(0.525)	
$\Delta TotRisk_t^{*}$			-0.589	
			(0.756)	
Adjusted R^2	0.008	0.008	0.004	0.011

Notes: Regressions of 6-month CIP deviations (b.p.) on short-maturity dollar-trade returns and relative total risk (p.p.), all in first differences. Newey and West (1987) (6 lags) standard errors. Sample: 2000:01–2021:03.

vestors active in short-lived and long-lived asset markets appear distinct (see, e.g., Du et al., 2023). Appendix C details this extension of the model. Specifically, investors active in markets with long-lived assets such as long-maturity bonds and equities are characterized by SDF process $\{M_{t,t+1}\}$ (measured consistently using (2)), whereas investors active in markets with short bonds are characterized by $\{\hat{M}_{t,t+1}\}$.[28] Critically, in the environment we specify, the long-maturity equilibrium relationship is unaffected, but the short-maturity relationship is distorted, rationalizing the findings above.

In sum: over short maturities, cross-border convenience co-moves with dollar-trade returns, not with relative risk. Over long maturities, permanent risk differentials matter for convenience and the combination of rising relative permanent risk and declining convenience offsets to leave long-maturity dollar-trade returns, and therefore the U.S. dollar largely insulated.

6 Conclusion

In this paper we use asset prices to ascertain whether foreign investors have changed their valuation of USD and dollar assets relative to a panel of other advanced economies and, if so, for which dollar assets and at what maturities. Our analysis is motivated by striking patterns in the

[28] Appendix C also provides evidence that short-maturity CIP deviations are associated with intermediary leverage and the VIX, which likely correlate with the segmentation friction we highlight.

data which raise numerous questions. Namely, why do long-maturity U.S. bonds appear to have lost their 'specialness' relative to other comparable government bonds—a trend that started in the early 2000s—while excess returns on currency trades funded in USD have remained stable? How can U.S. equities deliver higher (expected) returns relative to other advanced economies (i.e., command higher risk premia), while short-maturity U.S. bonds continue to be considered particularly safe? Relying on a canonical two-country no-arbitrage framework, our analysis emphasizes the role of convenience yields as an equilibrium adjustment to risk, thus drawing equilibrium links between bonds, currency and equity markets.

We highlight a specific role for fluctuations in relative permanent (macro) risk—revisions to expectations about outcomes in the distant future—in explaining the decline in long-maturity convenience yields, over and above complementary channels operating through rising U.S. debt and intermediary leverage. Rising permanent risk is consistent with a 'scarring of beliefs' (Kozlowski et al., 2019), higher disaster risk (Farhi and Gourio, 2018), and secular trends in the aggregate profit share (Greenwald et al., 2025; Marin and Jung, 2026). Our results imply that stabilizing bond and currency markets requires tools that also address perceptions of long-run risk—whereas existing policies are traditionally aimed at short-term business-cycle objectives.

References

Acharya, V. V. and T. Laarits (2024): "When Do Treasuries Earn the Convenience Yield? — A Hedging Perspective," Working Paper 31863, National Bureau of Economic Research.

Adrian, T., R. K. Crump, and E. Moench (2013): "Pricing the term structure with linear regressions," *Journal of Financial Economics*, 110, 110–138.

Alvarez, F. and U. J. Jermann (2005): "Using asset prices to measure the persistence of the marginal utility of wealth," *Econometrica*, 73, 1977–2016.

Anderson, N. and J. Sleath (2001): "New estimates of the UK real and nominal yield curves," Bank of England Staff Working Papers, Bank of England.

Andrews, S., R. Colacito, M. M. Croce, and F. Gavazzoni (2024): "Concealed Carry," *Journal of Financial Economics*, 159, 103874.

Arvai, K. and N. Coimbra (2024): "Privilege Lost? The Rise and Fall of a Dominant Global Currency," CEPR Discussion Papers 19078, Centre for Economic Policy Research.

Atkeson, A., J. Heathcote, and F. Perri (2024): "There Is No Excess Volatility Puzzle," NBER Working Paper 32481, National Bureau of Economic Research.

——— (2025): "Reconciling Macroeconomics and Finance for the U.S. Corporate Sector: 1929 to Present," Working Paper 33459, National Bureau of Economic Research.

Augustin, P., M. Chernov, L. Schmid, and D. Song (2024): "The Term Structure of Covered Interest Rate Parity Violations," *The Journal of Finance*, 79, 2077–2114.

Backus, D., N. Boyarchenko, and M. Chernov (2018): "Term structures of asset prices and returns," *Journal of Financial Economics*, 129, 1–23.

Backus, D. K., S. Foresi, and C. I. Telmer (2001): "Affine term structure models and the forward premium anomaly," *The Journal of Finance*, 56, 279–304.

Backus, D. K. and G. W. Smith (1993): "Consumption and Real Exchange Rates in Dynamic Economies with Non-Traded Goods," *Journal of International Economics*, 35, 297–316.

Bakshi, G. and F. Chabi-Yo (2012): "Variance bounds on the permanent and transitory components of stochastic discount factors," *Journal of Financial Economics*, 105, 191–208.

Bansal, R. and B. N. Lehmann (1997): "Growth-Optimal Portfolio Restrictions On Asset Pricing Models," *Macroeconomic Dynamics*, 1, 333–354.

Bianchi, J. and S. Bigio (2026): "Portfolio Choice and Settlement Frictions: A Theory of Endogenous Convenience Yields," *Journal of Economic Theory*, forthcoming; NBER Working Paper 34474.

Bianchi, J., S. Bigio, and C. Engel (2021): "Scrambling for Dollars: International Liquidity, Banks and Exchange Rates," NBER Working Paper 29457.

Brandt, M. W., J. H. Cochrane, and P. Santa-Clara (2006): "International risk sharing is better than you think, or exchange rates are too smooth," *Journal of Monetary Economics*, 53, 671–698.

Campbell, J. Y. (2008): "Viewpoint: Estimating the equity premium," *Canadian Journal of Economics/Revue canadienne d'économique*, 41, 1–21.

Campbell, J. Y., A. W. Lo, and A. C. MacKinlay (1997): *The Econometrics of Financial Markets*, Princeton, NJ: Princeton University Press.

Campbell, J. Y. and S. B. Thompson (2007): "Predicting Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average?" *The Review of Financial Studies*, 21, 1509–1531.

Caramp, N. and D. Silva (2025): "Monetary Policy and Wealth Effects: The Role of Risk and Heterogeneity," *Journal of Finance*, forthcoming.

Chabi-Yo, F. and R. Colacito (2019): "The term structures of coentropy in international financial markets," *Management Science*, 65, 3541–3558.

Chinn, M. D. and G. Meredith (2005): "Testing Uncovered Interest Parity at Short and Long Horizons during the Post-Bretton Woods Era," NBER Working Papers 11077, National Bureau of Economic Research, Inc.

COLACITO, R. AND M. M. CROCE (2011): "Risks for the long run and the real exchange rate," *Journal of Political economy*, 119, 153–181.

CONSTANTINIDES, G. M. (2002): "Rational Asset Prices," *The Journal of Finance*, 57, 1567–1591.

CORSETTI, G., L. DEDOLA, AND S. LEDUC (2008): "International risk sharing and the transmission of productivity shocks," *The Review of Economic Studies*, 75, 443–473.

COX, J. C., J. E. INGERSOLL, AND S. A. ROSS (1985): "The Theory of the Term Structure of Interest Rates," *Econometrica*, 53, 385–407.

DE LA'O, R. AND S. MYERS (2021): "Subjective Cash Flow and Discount Rate Expectations," *The Journal of Finance*, 76, 1339–1387.

DIAMOND, W. AND P. VAN TASSEL (2022): "Risk-Free Rates and Convenience Yields Around the World," Staff Reports 1032, Federal Reserve Bank of New York.

DICKEY, D. A. AND W. A. FULLER (1979): "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Association*, 74, 427–431.

DISON, W. AND A. RATTAN (2017): "An Improved Model for Understanding Equity Prices," *Bank of England Quarterly Bulletin*, 57, 86–96.

DU, W., B. M. HÉBERT, AND W. LI (2023): "Intermediary Balance Sheets and the Treasury Yield Curve," *Journal of Financial Economics*, 150, available as Art. 103722.

DU, W., J. IM, AND J. SCHREGER (2018a): "The U.S. Treasury Premium," *Journal of International Economics*, 112, 167–181.

DU, W., R. KEERATI, AND J. SCHREGER (2026): "Decoupling Dollar and Treasury Privilege," NBER Working Paper 35000, National Bureau of Economic Research, Cambridge, MA.

DU, W., A. TEPPER, AND A. VERDELHAN (2018b): "Deviations from covered interest rate parity," *The Journal of Finance*, 73, 915–957.

ENGEL, C. (2014): "Exchange Rates and Interest Parity," Elsevier, vol. 4, chap. Chapter 8, 453–522.

ENGEL, C. AND S. P. Y. WU (2023): "Liquidity and Exchange Rates: An Empirical Investigation," *The Review of Economic Studies*, 90, 2395–2438.

ENGLE, R. AND C. GRANGER (1987): "Co-integration and Error Correction: Representation, Estimation, and Testing," *Econometrica*, 55, 251–76.

FARHI, E. AND X. GABAIX (2016): "Rare disasters and exchange rates," *The Quarterly Journal of Economics*, 131, 1–52.

FARHI, E. AND F. GOURIO (2018): "Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia," *Brookings Papers on Economic Activity*, 49, 147–250.

GABAIX, X. AND M. MAGGIORI (2015): "International Liquidity and Exchange Rate Dynamics," *The Quarterly Journal of Economics*, 130, 1369–1420.

GORDON, M. J. (1962): "The savings investment and valuation of a corporation," *The Review of Economics and Statistics*, 37–51.

GOURINCHAS, P.-O., W. D. RAY, AND D. VAYANOS (2025): "A Preferred-Habitat Model of Term Premia, Exchange Rates, and Monetary Policy Spillovers," *American Economic Review*, 115, 3788–3824.

GOURINCHAS, P.-O. AND H. REY (2007): "International financial adjustment," *Journal of Political Economy*, 115, 665–703.

GOURINCHAS, P.-O., H. REY, AND N. GOVILLOT (2010): "Exorbitant Privilege and Exorbitant Duty," IMES Discussion Paper Series 10-E-20, Institute for Monetary and Economic Studies, Bank of Japan.

GREENWALD, D. L., M. LETTAU, AND S. C. LUDVIGSON (2025): "How the Wealth Was Won: Factor Shares as Market Fundamentals," *Journal of Political Economy*, 133, 1083–1132.

GÜRKAYNAK, R. S., B. SACK, AND J. H. WRIGHT (2007): "The US Treasury yield curve: 1961 to the present," *Journal of monetary Economics*, 54, 2291–2304.

HANSEN, L. P. AND R. JAGANNATHAN (1991): "Implications of security market data for models of dynamic economies," *Journal of political economy*, 99, 225–262.

HANSEN, L. P. AND J. A. SCHEINKMAN (2009): "Long-Term Risk: An Operator Approach," *Econometrica*, 77, 177–234.

HE, Z., B. KELLY, AND A. MANELA (2017): "Intermediary asset pricing: New evidence from many asset classes," *Journal of Financial Economics*, 126, 1–35.

HE, Z., S. NAGEL, AND Z. SONG (2022): "Treasury inconvenience yields during the COVID-19 crisis," *Journal of Financial Economics*, 143, 57–79.

ITSKHOKI, O. AND D. MUKHIN (2021): "Exchange rate disconnect in general equilibrium," *Journal of Political Economy*, 129, 2183–2232.

JEANNE, O. AND A. K. ROSE (2002): "Noise trading and exchange rate regimes," *The Quarterly Journal of Economics*, 117, 537–569.

JERMANN, U. (2020): "Negative swap spreads and limited arbitrage," *The Review of Financial Studies*, 33, 212–238.

JIANG, Z., A. KRISHNAMURTHY, AND H. LUSTIG (2021a): "Foreign safe asset demand and the dollar exchange rate," *The Journal of Finance*, 76, 1049–1089.

JIANG, Z., A. KRISHNAMURTHY, AND H. N. LUSTIG (2021b): "Beyond incomplete spanning: Convenience yields and exchange rate disconnect," *Available at SSRN 3852298*.

JIANG, Z. AND R. RICHMOND (2023): "Convenience Yields and Asset Pricing Models," *Working Paper*.

JIANG, Z., R. RICHMOND, AND T. ZHANG (2024): "Convenience Lost," *Working Paper*.

KEKRE, R. AND M. LENEL (2024): "The Flight to Safety and International Risk Sharing," *American Economic Review*, 114, 1650–91.

KOLLMANN, R. (1995): "Consumption, Real Exchange Rates and the Structure of International Asset Markets," *Journal of International Money and Finance*, 14, 191–211.

KOZLOWSKI, J., L. VELDKAMP, AND V. VENKATESWARAN (2019): "The Tail That Keeps the Riskless Rate Low," *NBER Macroeconomics Annual*, 33, 253–283.

KREMENS, L. AND I. MARTIN (2019): "The Quanto Theory of Exchange Rates," *American Economic Review*, 109, 810–843.

KRISHNAMURTHY, A. AND H. N. LUSTIG (2019): "Mind the gap in sovereign debt markets: The US Treasury basis and the dollar risk factor," in *2019 Jackson Hole Economic Symposium, Stanford University Graduate School of Business Research Paper*, 3443231.

KRISHNAMURTHY, A. AND A. VISSING-JORGENSEN (2012): "The aggregate demand for treasury debt," *Journal of Political Economy*, 120, 233–267.

LETTAU, M. AND S. LUDVIGSON (2001): "Consumption, Aggregate Wealth, and Expected Stock Returns," *Journal of Finance*, 56, 815–849.

LIAO, G. Y. AND T. ZHANG (2025): "The Hedging Channel of Exchange Rate Determination," *The Review of Financial Studies*, 38, 1–38.

LIU, Y., L. SCHMID, AND A. YARON (2020): "The risks of safe assets," *Available at SSRN 3699618*.

LLOYD, S. AND E. MARIN (2020): "Exchange rate risk and business cycles," Bank of England Staff Working Papers 872, Bank of England.

LUSTIG, H., N. ROUSSANOV, AND A. VERDELHAN (2011): "Common Risk Factors in Currency Markets," *Review of Financial Studies*, 24, 3731–3777.

LUSTIG, H., A. STATHOPOULOS, AND A. VERDELHAN (2019): "The term structure of currency carry trade risk premia," *American Economic Review*, 109, 4142–77.

LUSTIG, H. AND A. VERDELHAN (2019): "Does incomplete spanning in international financial markets help to explain exchange rates?" *American Economic Review*, 109, 2208–44.

MACKINNON, J. G. (2010): "Critical Values For Cointegration Tests," Working Paper 1227, Economics Department, Queen's University.

MAGGIORI, M. (2017): "Financial intermediation, international risk sharing, and reserve currencies," *American Economic Review*, 107, 3038–71.

MARIN, E. AND S. R. SINGH (2026): "Incomplete Markets and Exchange Rates," SSRN Working Paper 6169686, Social Science Research Network.

MARIN, E. A. AND J. Y. JUNG (2026): "Looking for Risk: Asset Pricing Bounds and the Wealth Share," Working Paper.

Martin, I. (2017): "What is the Expected Return on the Market?" *The Quarterly Journal of Economics*, 132, 367–433.

Newey, W. and K. West (1987): "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55, 703–08.

Pagan, A. (1984): "Econometric Issues in the Analysis of Regressions with Generated Regressors," *International Economic Review*, 25, 221–247.

Pavlova, A. and R. Rigobon (2007): "Asset Prices and Exchange Rates," *Review of Financial Studies*, 20, 1139–1180.

——— (2008): "The Role of Portfolio Constraints in the International Propagation of Shocks," *Review of Economic Studies*, 75, 1215–1256.

Phillips, P. C. B. and B. E. Hansen (1990): "Statistical Inference in Instrumental Variables Regression with I(1) Processes," *Review of Economic Studies*, 57, 99–125.

Saikkonen, P. (1991): "Asymptotically Efficient Estimation of Cointegration Regressions," *Econometric Theory*, 7, 1–21.

Stock, J. H. (1987): "Asymptotic Properties of Least Squares Estimators of Cointegrating Vectors," *Econometrica*, 55, 1035–1056.

Stock, J. H. and M. W. Watson (1993): "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems," *Econometrica*, 61, 783–820.

Sun, T.-S. (1992): "Real and Nominal Interest Rates: A Discrete-Time Model and Its Continuous-Time Limit," *Journal of Financial Economics*, 30, 125–152.

Theil, H. (1967): *Economics and Information Theory*, Amsterdam: North-Holland Publishing Company.

Valchev, R. (2020): "Bond convenience yields and exchange rate dynamics," *American Economic Journal: Macroeconomics*, 12, 124–66.

Vayanos, D. and J.-L. Vila (2021): "A Preferred-Habitat Model of the Term Structure of Interest Rates," *Econometrica*, 89, 77–112.

Verdelhan, A. (2010): "A Habit-Based Explanation of the Exchange Rate Risk Premium," *Journal of Finance*, 65, 123–146.

Wright, J. (2011): "Term Premia and Inflation Uncertainty: Empirical Evidence from an International Panel Dataset," *American Economic Review*, 101, 1514–34.

Appendix

A Model Proofs and Derivations

A.1 Deriving the Exchange-Rate Process

Combining (5) and (7), evaluating at $k = 1$, and expanding yields:

$$\mathbb{E}_t[\Delta e_{t+1}] + \mathcal{L}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) + \mathcal{C}_t\left(M_{t,t+1}, \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) + r_t^{*(1)} - r_t^{(1)} + \theta_t^{H,F(1)} - \theta_t^{H,H(1)} = 0 \quad \text{(A.1)}$$

Then, using (6) and (8):

$$-\mathbb{E}_t[\Delta e_{t+1}] + \mathcal{L}_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) + \mathcal{C}_t\left(M^*_{t,t+1}, \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) + r_t^{(1)} - r_t^{*(1)} + \theta_t^{F,H(1)} - \theta_t^{F,F(1)} = 0 \quad \text{(A.2)}$$

Combining the above yields (9).

A.2 Proof to Proposition 1

Taking expectations of the exchange-rate process (10), and substituting in for $M_{t,t+1}$ and $M^*_{t,t+1}$ using the log-entropy expansions of the $k = 1$-period domestic Euler equations (5) and (6), respectively, we can write:

$$\mathbb{E}_t[\Delta e_{t+1}] = -r_t^* - \theta_t^{F,F(1)} - \mathcal{L}_t(M^*_{t,t+1}) + r_t + \theta_t^{H,H(1)} + \mathcal{L}_t(M_{t,t+1}) + \mathbb{E}_t[\eta_{t+1}]$$

Substituting (11), applying the definition of the *ex post* excess currency return (15), and rearranging yields the result. □

A.3 Proof to Lemma 1

Start by rewriting the Home Euler equation for the Home k-period bond (5):

$$\begin{aligned}
e^{-\theta_t^{H,H(k)}} =& \mathbb{E}_t\left[M_{t,t+k}R_t^{(k)}\right] \\
e^{-\theta_t^{H,H(k)}} =& \mathbb{E}_t\left[\frac{\Lambda_{t+k}}{\Lambda_t}\frac{1}{P_t^{(k)}}\right] \\
\Rightarrow\ P_t^{(k)} =& \mathbb{E}_t\left[\frac{\Lambda_{t+k}}{\Lambda_t}\right]e^{\theta_t^{H,H(k)}}
\end{aligned}$$

so, also $P_{t+1}^{(k-1)} = \mathbb{E}_{t+1}\left[\frac{\Lambda_{t+k}}{\Lambda_{t+1}}\right] e^{\theta_{t+1}^{H,H(k-1)}}$.

Now, solve for the one-period holding return on a long-term bond, following similar steps to Alvarez and Jermann (2005) in their proof to Proposition 2(i):

$$\begin{aligned}
R_{t,t+1}^{(\infty)} &\equiv \lim_{k\to\infty} R_{t,t+1}^{(k)} \\
&= \lim_{k\to\infty} \frac{P_{t+1}^{(k-1)}}{P_t^{(k)}} \\
&= \lim_{k\to\infty} \frac{e^{\theta_{t+1}^{H,H(k-1)}} \cdot \mathbb{E}_{t+1}\left[\frac{\Lambda_{t+k}}{\Lambda_{t+1}}\right]}{e^{\theta_t^{H,H(k)}} \cdot \mathbb{E}_t\left[\frac{\Lambda_{t+k}}{\Lambda_t}\right]} \\
&= \frac{\lim_{k\to\infty} e^{\theta_{t+1}^{H,H(k-1)}} \cdot \frac{\mathbb{E}_{t+1}[\Lambda_{t+k}]/\beta^{t+k}}{\Lambda_{t+1}}}{\lim_{k\to\infty} e^{\theta_t^{H,H(k)}} \cdot \frac{\mathbb{E}_t[\Lambda_{t+k}]/\beta^{t+k}}{\Lambda_t}} \\
&= e^{-\theta_{t,t+1}^{H,H(\infty)}} \frac{\lim_{k\to\infty} \frac{\mathbb{E}_{t+1}[\Lambda_{t+k}]/\beta^{t+k}}{\Lambda_{t+1}}}{\lim_{k\to\infty} \frac{\mathbb{E}_t[\Lambda_{t+k}]/\beta^{t+k}}{\Lambda_t}} \\
&= e^{-\theta_{t,t+1}^{H,H(\infty)}} \frac{\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_{t+1}}}{\frac{\Lambda_t^{\mathbb{P}}}{\Lambda_t}} \\
&= e^{-\theta_{t,t+1}^{H,H(\infty)}} \frac{\Lambda_t^{\mathbb{T}}}{\Lambda_{t+1}^{\mathbb{T}}},
\end{aligned}$$

where line 1 is a definition, line 2 uses definition for holding-period returns, line 3 uses bond-pricing and Euler equations, line 4 rearranges and multiplies and divides by β^{t+k}, line 5 defines convenience holding-period return, line 6 uses the definition of $\Lambda^{\mathbb{P}}$, and line 7 uses the definition of the pricing kernel $\Lambda = \Lambda^{\mathbb{P}}\Lambda^{\mathbb{T}}$. Rearranging this final expression yields the result in equation (18). □

A.4 Proof to Lemma 2

Taking logs of (13):

$$\log \mathbb{E}_t\left[M_{t,t+1}R_{t,t+1}^g\right] = 0$$

By concavity of the log, we have that:

$$\begin{aligned}
\log \mathbb{E}_t\left[M_{t,t+1}R_{t,t+1}^g\right] &= 0 \geq \mathbb{E}_t \log\left[M_{t,t+1}R_{t,t+1}^g\right] \\
\Rightarrow -\mathbb{E}_t \log M_{t,t+1} &\geq \mathbb{E}_t \log[R_{t,t+1}^g]
\end{aligned}$$

Using this in the definition of the entropy measure, we can derive:

$$
\begin{aligned}
\mathcal{L}_t(M_{t,t+1}) &= \log \mathbb{E}_t[M_{t,t+1}] - \mathbb{E}_t \log M_{t,t+1} \\
\mathcal{L}_t(M_{t,t+1}) &\geq \log \mathbb{E}_t[M_{t,t+1}] + \mathbb{E}_t \log[R^g_{t,t+1}] \\
\mathcal{L}_t(M_{t,t+1}) &\geq \log \left[\frac{1}{R^{(1)}_t} e^{-\theta^{H,H(1)}_t}\right] + \mathbb{E}_t \log[R^g_{t,t+1}] \\
\mathcal{L}_t(M_{t,t+1}) &\geq \mathbb{E}_t \log \left[\frac{R^g_{t,t+1}}{R_t}\right] - \theta^{H,H(1)}_t
\end{aligned}
$$

where line 1 is the definition of entropy, line 2 follows from the inequality derived above, line 3 uses the Home Euler for a Home one-period bond (5), and line 4 rearranges. This verifies (19).

Next, decompose total risk starting with the definition of conditional entropy $\mathcal{L}_t(\cdot)$:

$$
\begin{aligned}
\mathcal{L}_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) &= \log \mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\right] - \mathbb{E}_t \log \frac{\Lambda_{t+1}}{\Lambda_t} \\
&= \log\left(P^{(1)}_t e^{-\theta^{H,H(1)}_t}\right) - \mathbb{E}_t \log \frac{\Lambda^{\mathbb{P}}_{t+1}\Lambda^{\mathbb{T}}_{t+1}}{\Lambda^{\mathbb{P}}_t\Lambda^{\mathbb{T}}_t} \\
&= \log\left(P^{(1)}_t e^{-\theta^{H,H(1)}_t}\right) - \mathbb{E}_t \log \frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t} - \mathbb{E}_t \log \frac{\Lambda^{\mathbb{T}}_{t+1}}{\Lambda^{\mathbb{T}}_t} \\
&= \log\left(P^{(1)}_t e^{-\theta^{H,H(1)}_t}\right) + \mathcal{L}_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) - \log \mathbb{E}_t \frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t} - \mathbb{E}_t \log \frac{\Lambda^{\mathbb{T}}_{t+1}}{\Lambda^{\mathbb{T}}_t} \\
&= \log\left(\frac{1}{R^{(1)}_t} e^{-\theta^{H,H(1)}_t}\right) + \mathcal{L}_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) - \mathbb{E}_t \log\left(e^{-\theta^{H,H(\infty)}_{t,t+1}} \cdot \frac{1}{R^{(\infty)}_{t,t+1}}\right) \\
&= -\log R^{(1)}_t - \theta^{H,H(1)}_t + \mathcal{L}_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) + \mathbb{E}_t\theta^{H,H(\infty)}_{t,t+1} + \mathbb{E}_t \log R^{(\infty)}_{t,t+1} \\
&= \mathbb{E}_t \log \frac{R^{(\infty)}_{t,t+1}}{R^{(1)}_t} + \mathcal{L}_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) - \theta^{H,H(1)}_t + \mathbb{E}_t\theta^{H,H(\infty)}_{t,t+1}
\end{aligned} \tag{A.3}
$$

where line 1 is a definition, line 2 uses the pricing expression and the definition of the pricing kernel, line 3 separates the second term, line 4 uses the definition $\mathcal{L}_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) = \log \mathbb{E}_t \frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t} - \mathbb{E}_t \log \frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}$, line 5 uses the facts that $P^{(1)}_t = 1/R^{(1)}_t$, $\log \mathbb{E}_t \frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t} = 0$ and (from Lemma 1) $\frac{\Lambda^{\mathbb{T}}_{t+1}}{\Lambda^{\mathbb{T}}_t} = e^{-\theta^{H,H(\infty)}_{t,t+1}} \cdot \frac{1}{R^{(\infty)}_{t+1}}$, line 6 rearranges, line 7 concludes. Rearranging this final expression, and using definitions of SDFs and excess returns, yields the result.

Finally, this can be written as:

$$\mathcal{L}_t(M^{\mathbb{P}}_{t,t+1}) + \mathbb{E}_t[rx^{(\infty)}_{t+1}] - \theta_t^{H,H(1)} + \mathbb{E}_t[\theta_{t,t+1}^{H,H(\infty)}] \geq \mathbb{E}_t \log\left[\frac{R^g_{t,t+1}}{R_t}\right] - \theta_t^{H,H(1)}$$
$$\mathcal{L}_t(M^{\mathbb{P}}_{t,t+1}) + \mathbb{E}_t[\theta_{t,t+1}^{H,H(\infty)}] \geq \mathbb{E}_t \log\left[\frac{R^g_{t,t+1}}{R_t}\right] - \mathbb{E}_t[rx^{(\infty)}_{t+1}]$$

where line 1 substitutes and line 2 rearranges to yield condition (20). □

A.5 Proof to Lemma 3

The k-period Home-on-Home and Foreign-on-Home Euler equations, evaluated with the k-period exchange-rate process $\mathcal{E}_{t+k}/\mathcal{E}_t = (M^*_{t,t+k}/M_{t,t+k})\, e^{\eta^{(k)}}$ where $\eta^{(k)} \equiv \sum_{\tau=1}^{k} \eta_{t+\tau}$, are given by $\mathbb{E}_t[M_{t,t+k}]\, R_t^{(k)} = e^{-\theta_t^{H,H(k)}}$ and $\mathbb{E}_t[M_{t,t+k} e^{-\eta^{(k)}}]\, R_t^{(k)} = e^{-\theta_t^{F,H(k)}}$. Dividing eliminates the bond return:

$$\theta_t^{F,H(k)} - \theta_t^{H,H(k)} = \log \mathbb{E}_t[M_{t,t+k}] - \log \mathbb{E}_t\left[M_{t,t+k}\, e^{-\eta^{(k)}}\right]. \tag{A.4}$$

In turn,

$$\mathbb{E}_t\left[M_{t,t+k}\, e^{-\eta^{(k)}}\right] = \mathbb{E}_t\Big[M_{t,t+1}\, e^{-\eta_{t+1}}\, \mathbb{E}_{t+1}\left[M_{t+1,t+k}\, e^{-\eta_{t+1,t+k}}\right]\Big]. \tag{A.5}$$

Applying (A.4), evaluated at $t+1$ for maturity $k-1$, to the inner expectation,

$$\mathbb{E}_{t+1}\left[M_{t+1,t+k}\, e^{-\eta_{t+1,t+k}}\right] = \mathbb{E}_{t+1}\left[M_{t+1,t+k}\right] e^{-(\theta_{t+1}^{F,H(k-1)} - \theta_{t+1}^{H,H(k-1)})}. \tag{A.6}$$

Substituting (A.6) into (A.5), and using the law of iterated expectations, (A.4) simplifies:

$$\theta_t^{F,H(k)} - \theta_t^{H,H(k)} = \log \mathbb{E}_t[M_{t,t+k}] - \log \mathbb{E}_t\Big[M_{t,t+k}\, e^{-\eta_{t+1} - (\theta_{t+1}^{F,H(k-1)} - \theta_{t+1}^{H,H(k-1)})}\Big]. \tag{A.7}$$

For any positive random variable Z, the definition of conditional entropy gives $\log \mathbb{E}_t[Z] = \mathcal{L}_t(Z) + \mathbb{E}_t[\log Z]$. Apply this to both expectations on the right-hand side of (A.7):

$$\theta_t^{F,H(k)} - \theta_t^{H,H(k)} = \mathcal{L}_t(M_{t,t+k}) - \mathcal{L}_t\Big(M_{t,t+k}\, e^{-\eta_{t+1} - (\theta_{t+1}^{F,H(k-1)} - \theta_{t+1}^{H,H(k-1)})}\Big) + \mathbb{E}_t[\eta_{t+1}] + \mathbb{E}_t\left[\theta_{t+1}^{F,H(k-1)} - \theta_{t+1}^{H,H(k-1)}\right] \tag{A.8}$$

For the one-period wedge $\mathbb{E}_t[\eta_{t+1}]$, substitute the $k=1$ identity,

$$\mathbb{E}_t[\eta_{t+1}] = \big(\theta_t^{F,H(1)} - \theta_t^{H,H(1)}\big) + \mathcal{L}_t(e^{-\eta_{t+1}}) + \mathcal{C}_t(M_{t,t+1}, e^{-\eta_{t+1}}) \tag{A.9}$$

Inserting (A.9) into (A.8) splits off the one-period basis and leaves a four-term remainder, which is $\Phi_t^{H(k)}$:

$$\theta_t^{F,H(k)} - \theta_t^{H,H(k)} = \big(\theta_t^{F,H(1)} - \theta_t^{H,H(1)}\big) + \mathbb{E}_t\big[\theta_{t+1}^{F,H(k-1)} - \theta_{t+1}^{H,H(k-1)}\big] + \Phi_t^{H(k)}, \tag{A.10}$$

where:

$$\Phi_t^{H(k)} = \mathcal{L}_t(M_{t,t+k}) - \mathcal{L}_t\Big(M_{t,t+k}\, e^{-\eta_{t+1}-(\theta_{t+1}^{F,H(k-1)}-\theta_{t+1}^{H,H(k-1)})}\Big) + \mathcal{L}_t(e^{-\eta_{t+1}}) + \mathcal{C}_t(M_{t,t+1}, e^{-\eta_{t+1}}). \tag{A.11}$$

Moving the term $\mathbb{E}_t\big[\theta_{t+1}^{F,H(k-1)} - \theta_{t+1}^{H,H(k-1)}\big]$ in (A.10) to the left-hand side gives the holding-period convenience-yield differential of Lemma 3. The Foreign-bond relation (22) follows from similar steps applied to the Home-on-Foreign and Foreign-on-Foreign Euler equations, and is given by

$$\Phi_t^{F(k)} = \mathcal{L}_t\big(M^*_{t,t+k}\, e^{\eta_{t+1}+(\theta_{t+1}^{F,F(k-1)}-\theta_{t+1}^{H,F(k-1)})}\big) - \mathcal{L}_t\big(M^*_{t,t+k}\big) - \mathcal{L}_t\big(e^{\eta_{t+1}}\big) - \mathcal{C}_t\big(M^*_{t,t+1},\, e^{\eta_{t+1}}\big). \tag{A.12}$$

□

A.6 Proof to Proposition 2

Take the decomposition of total risk from equation (A.3) from Lemma 2:

$$\mathcal{L}_t(M_{t,t+1}) = \mathcal{L}_t(M^{\mathbb{P}}_{t,t+1}) + \mathbb{E}_t[rx^{(\infty)}_{t+1}] - \theta_t^{H,H(1)} + \mathbb{E}_t[\theta_{t,t+1}^{H,H(\infty)}]$$

and its Foreign counterpart. Substituting these into Proposition 1 yields:

$$\begin{aligned}\mathbb{E}_t[rx^{FX}_{t+1}] &= \mathcal{L}_t(M^{\mathbb{P}}_{t,t+1}) + \mathbb{E}_t[rx^{(\infty)}_{t+1}] - \theta_t^{H,H(1)} + \mathbb{E}_t[\theta_{t,t+1}^{H,H(\infty)}] - \mathcal{L}_t(M^{\mathbb{P}*}_{t,t+1}) - \mathbb{E}_t[rx^{(\infty)*}_{t+1}] \\ &\quad + \theta_t^{F,F(1)} - \mathbb{E}_t[\theta_{t,t+1}^{F,F(\infty)}] + \theta_t^{F,H(1)} - \theta_t^{F,F(1)} + \mathcal{L}_t(e^{-\eta_{t+1}}) + \mathcal{C}_t(M_{t,t+1}, e^{-\eta_{t+1}})\end{aligned}$$

$$\mathbb{E}_t[rx_{t+1}^{CT,(\infty)}] = \mathcal{L}_t(M_{t,t+1}^{\mathbb{P}}) - \mathcal{L}_t(M_{t,t+1}^{\mathbb{P}*}) - \theta_t^{H,H(1)} + \mathbb{E}_t[\theta_{t,t+1}^{H,H(\infty)}] - \mathbb{E}_t[\theta_{t,t+1}^{F,F(\infty)}]$$
$$+ \theta_t^{F,H(1)} + \mathcal{L}_t(e^{-\eta_{t+1}}) + \mathcal{C}_t(M_{t,t+1}, e^{-\eta_{t+1}})$$

where line 2 rearranges, cancels like terms, and uses the definition of $\mathbb{E}_t[rx_{t+1}^{CT,(\infty)}]$.

Assuming standard regularity conditions, evaluating Lemma 3 at $k = \infty$:

$$\mathbb{E}_t[\theta_{t,t+1}^{H,H(\infty)}] = \mathbb{E}_t[\theta_{t,t+1}^{F,H(\infty)}] - (\theta_t^{F,H(1)} - \theta_t^{H,H(1)}) - \Phi_t^{H(\infty)}.$$

Now, substitute $\mathbb{E}_t[\theta_{t,t+1}^{H,H(\infty)}]$ into the expression for $\mathbb{E}_t[rx_{t+1}^{CT,(\infty)}]$:

$$\begin{aligned}
\mathbb{E}_t[rx_{t+1}^{CT,(\infty)}] &= \mathcal{L}_t(M_{t,t+1}^{\mathbb{P}}) - \mathcal{L}_t(M_{t,t+1}^{\mathbb{P}*}) - \theta_t^{H,H(1)} + \mathbb{E}_t[\theta_{t,t+1}^{H,H(\infty)}] - \mathbb{E}_t[\theta_{t,t+1}^{F,F(\infty)}] \\
&\quad + \theta_t^{F,H(1)} + \mathcal{L}_t(e^{-\eta_{t+1}}) + \mathcal{C}_t(M_{t,t+1}, e^{-\eta_{t+1}}) \\
&= \mathcal{L}_t(M_{t,t+1}^{\mathbb{P}}) - \mathcal{L}_t(M_{t,t+1}^{\mathbb{P}*}) - \theta_t^{H,H(1)} + \mathbb{E}_t[\theta_{t,t+1}^{F,H(\infty)}] - (\theta_t^{F,H(1)} - \theta_t^{H,H(1)}) - \Phi_t^{H(\infty)} \\
&\quad - \mathbb{E}_t[\theta_{t,t+1}^{F,F(\infty)}] + \theta_t^{F,H(1)} + \mathcal{L}_t(e^{-\eta_{t+1}}) + \mathcal{C}_t(M_{t,t+1}, e^{-\eta_{t+1}}) \\
&= \mathcal{L}_t(M_{t,t+1}^{\mathbb{P}}) - \mathcal{L}_t(M_{t,t+1}^{\mathbb{P}*}) + \mathbb{E}_t[\theta_{t,t+1}^{F,H(\infty)} - \theta_{t,t+1}^{F,F(\infty)}] - \Omega_t
\end{aligned}$$

where line 1 restates the expression, line 2 substitutes Lemma 3, and line 3 cancels the one-period convenience yields, $-\theta_t^{H,H(1)} - (\theta_t^{F,H(1)} - \theta_t^{H,H(1)}) + \theta_t^{F,H(1)} = 0$, and collects the remaining terms into

$$\Omega_t \equiv \Phi_t^{H(\infty)} - \mathcal{L}_t(e^{-\eta_{t+1}}) - \mathcal{C}_t(M_{t,t+1}, e^{-\eta_{t+1}}),$$

yielding equation (23) in Proposition 2. □

A.7 Solving for the Incomplete Markets Wedge in Section 3.3

We solve for η_{t+1}, extending the methodology of Lustig and Verdelhan (2019); Marin and Singh (2026) to convenience yields. Define:

$$\delta^j \equiv (\pi_j^{F,H} - \pi_j^{H,H}) - (\pi_j^{F,F} - \pi_j^{H,F}) \qquad j \in \{\mathbb{P}, \mathbb{P}*\}. \tag{A.13}$$

Applying the conditional variance operator on the exchange-rate process $\Delta e_{t,t+1} = m^*_{t,t+1} - \widehat{m}_{t,t+1} + \eta_{t+1}$ and substituting $\mathrm{Cov}_t(m^*_{t,t+1} - m_{t,t+1}, \eta_{t+1})$ from (11)-(12), we obtain:

$$\mathrm{Var}_t(\eta_{t+1}) \;=\; \mathrm{Var}_t(m^*_{t,t+1} - m_{t,t+1}) \;-\; \mathrm{Var}_t(\Delta e_{t+1}) \;-\; 2 \sum_{j\in\{\mathbb{P},\mathbb{P}*\}} \delta^j z^j_t. \tag{A.14}$$

Let J_H and J_F collect the Home and Foreign risk factors the wedge spans, and write

$$\eta_{t+1} = \sum_{j\in J_H\cup J_F} \big[\psi^j z^j_t + \Gamma^j_1\sqrt{z^j_t}\, u^j_{t+1} + \Gamma^j_2\sqrt{z^j_t}\, \epsilon^j_{t+1}\big]. \tag{A.15}$$

By (A.18), $(\Gamma^j_1)^2 + (\Gamma^j_2)^2 = \gamma^j - \kappa^j - 2\delta^j$, so

$$\mathcal{L}_t(e^{-\eta_{t+1}}) = \tfrac{1}{2}\mathrm{Var}_t\eta_{t+1} = \sum_{j\in J_H\cup J_F} \big[(1-\xi^j)\,\mathcal{L}_t(M^j) - \delta^j z^j_t\big], \tag{A.16}$$

$$\mathcal{C}_t(M, e^{-\eta_{t+1}}) = -\mathrm{Cov}_t(m,\eta) = \sum_{j\in J_H} \big[-2(1-\xi^j)\,\mathcal{L}_t(M^j) + \delta^j z^j_t\big], \tag{A.17}$$

Substituting the SDF and denoting $\mathrm{Var}_t(\Delta e_{t+1}) = \kappa^{\mathbb{P}} z^{\mathbb{P}}_t + \kappa^{\mathbb{P}*} z^{\mathbb{P}*}_t$, gathering coefficients on $z^{\mathbb{P}}_t$, $z^{\mathbb{P}*}_t$ yields:

$$\underbrace{\big[(\Gamma^j_1)^2 + (\Gamma^j_2)^2\big] z^j_t}_{\mathrm{Var}_t(\eta_{t+1})} \;=\; \underbrace{\gamma^j z^j_t}_{\mathrm{Var}_t(m^*_{t+1} - m_{t,t+1})} \;-\; \underbrace{\kappa^j z^j_t}_{\mathrm{Var}_t(\Delta e_{t+1})} \;-\; 2\delta^j z^j_t, \qquad j\in\{\mathbb{P},\mathbb{P}*\}. \tag{A.18}$$

Solving for the coefficients yields:

$$\Gamma^{\mathbb{P}}_1 = \pm\sqrt{\gamma^{\mathbb{P}} - 2\delta^{\mathbb{P}} - \lambda^{\mathbb{P}}}, \qquad \Gamma^{\mathbb{P}*}_1 = \pm\sqrt{\gamma^{\mathbb{P}*} - 2\delta^{\mathbb{P}*} - \lambda^{\mathbb{P}*}}, \tag{A.19}$$

$$\Gamma^{\mathbb{P}}_2 = \pm\sqrt{\lambda^{\mathbb{P}} - \kappa^{\mathbb{P}}}, \qquad \Gamma^{\mathbb{P}*}_2 = \pm\sqrt{\lambda^{\mathbb{P}*} - \kappa^{\mathbb{P}*}}. \tag{A.20}$$

where λ^j is a parameter allocating the wedge's conditional variance between spanned and unspanned innovations

To keep all roots real, we require:

$$\begin{aligned} \gamma^{\mathbb{P}} - 2\delta^{\mathbb{P}} \;&\geq\; \lambda^{\mathbb{P}} \;\geq\; \kappa^{\mathbb{P}}, \\ \gamma^{\mathbb{P}*} - 2\delta^{\mathbb{P}*} \;&\geq\; \lambda^{\mathbb{P}*} \;\geq\; \kappa^{\mathbb{P}*}. \end{aligned} \tag{A.21}$$

We then match coefficients on each z^j in (11)-(12). For the Home factor $\mathbb{P}$, $\mathrm{Cov}_t(m^*, \eta)$ carries no $z^{\mathbb{P}}$ loading, so (12) at $z^{\mathbb{P}}$ pins the drift directly and the difference (11) − (12) pins the spanned loading in terms of the wedge variance and the convenience-yield-process loadings; for the Foreign factor $\mathbb{P}*$, $\mathrm{Cov}_t(m, \eta)$ carries no $z^{\mathbb{P}*}$ loading and the roles of (11) and (12) interchange. The drifts are given by:

$$\psi^{\mathbb{P}} = -\tfrac{1}{2}\big(\gamma^{\mathbb{P}} - \kappa^{\mathbb{P}} - 2\delta^{\mathbb{P}}\big) + (\pi^{F,F}_{\mathbb{P}} - \pi^{H,F}_{\mathbb{P}}), \tag{A.22}$$

$$\psi^{\mathbb{P}*} = \tfrac{1}{2}\big(\gamma^{\mathbb{P}*} - \kappa^{\mathbb{P}*} - 2\delta^{\mathbb{P}*}\big) + (\pi^{F,H}_{\mathbb{P}*} - \pi^{H,H}_{\mathbb{P}*}), \tag{A.23}$$

Substituting back into (11)-(12) yields:

$$\kappa^{\mathbb{P}} = \gamma^{\mathbb{P}} - \delta^{\mathbb{P}} - \sqrt{\gamma^{\mathbb{P}}\big(\gamma^{\mathbb{P}} - 2\delta^{\mathbb{P}} - \lambda^{\mathbb{P}}\big)} \ \geq \ 0, \tag{A.24}$$

$$\kappa^{\mathbb{P}*} = \gamma^{\mathbb{P}*} - \delta^{\mathbb{P}*} - \sqrt{\gamma^{\mathbb{P}*}\big(\gamma^{\mathbb{P}*} - 2\delta^{\mathbb{P}*} - \lambda^{\mathbb{P}*}\big)} \ \geq \ 0. \tag{A.25}$$

To keep all roots real, we take the signs consistent with (A.21). This pins the Home spanned loading to the negative root, $\Gamma^{\mathbb{P}}_1 = -\sqrt{\gamma^{\mathbb{P}} - 2\delta^{\mathbb{P}} - \lambda^{\mathbb{P}}}$ because the covariance of the kernel with the wedge is $\mathrm{Cov}_t\big(m_{t,t+1}, \eta_{t+1}\big) = -\sqrt{\gamma^{\mathbb{P}}}\,\Gamma^{\mathbb{P}}_1\, z^{\mathbb{P}}_t$, and only the negative root reproduces the spanned exchange-rate variance $\kappa^{\mathbb{P}}$ in (A.18). All remaining loadings, including the Foreign $\Gamma^{\mathbb{P}*}_1$, take the positive root.

A.8 Proof to Proposition 2'

By Proposition 2,

$$\mathbb{E}_t\big[rx^{CT,(\infty)}_{t+1}\big] = \mathcal{L}_t(M^{\mathbb{P}}_{t,t+1}) - \mathcal{L}_t(M^{\mathbb{P}*}_{t,t+1}) + \mathbb{E}_t\big[\theta^{F,H(\infty)}_{t,t+1} - \theta^{F,F(\infty)}_{t,t+1}\big] - \Omega_t, \tag{A.26}$$

where

$$\Omega_t = \Phi^{H(\infty)}_t - \mathcal{L}_t(e^{-\eta_{t+1}}) - \mathcal{C}_t(M_{t,t+1}, e^{-\eta_{t+1}}). \tag{A.27}$$

In the baseline the Home and Foreign kernels are purely permanent, $m = m^{\mathbb{P}}$ and $m^* = m^{\mathbb{P}*}$, and the wedge loads on the permanent factors $\{\mathbb{P}, \mathbb{P}*\}$ alone. We evaluate the above using the functional forms for SDFs and short-maturity convenience yields (24)-(26); direct computations give

$$\mathcal{L}_t(e^{-\eta_{t+1}}) = \tfrac{1}{2}\mathrm{Var}_t\eta_{t+1} = \tfrac{1}{2}\sum_{j\in\{\mathbb{P},\mathbb{P}*\}}\big[(\Gamma_1^j)^2 + (\Gamma_2^j)^2\big]\, z_t^j$$

$$= \tfrac{1}{2}\sum_{j\in\{\mathbb{P},\mathbb{P}*\}}(\gamma^j - \kappa^j - 2\delta^j)\, z_t^j = \sum_{j\in\{\mathbb{P},\mathbb{P}*\}}\big[(1-\xi^j)\,\mathcal{L}_t(M_{t,t+1}^j) - \delta^j z_t^j\big], \quad \text{(A.28)}$$

$$\mathcal{C}_t(M_{t,t+1}, e^{-\eta_{t+1}}) = -\mathrm{Cov}_t(m_{t,t+1}, \eta_{t+1}) = -\sqrt{\gamma^{\mathbb{P}}\big(\gamma^{\mathbb{P}} - 2\delta^{\mathbb{P}} - \lambda^{\mathbb{P}}\big)}\; z_t^{\mathbb{P}}$$

$$= -\big[\gamma^{\mathbb{P}}(1-\xi^{\mathbb{P}}) - \delta^{\mathbb{P}}\big]\, z_t^{\mathbb{P}} \;=\; -2(1-\xi^{\mathbb{P}})\,\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}}) + \delta^{\mathbb{P}} z_t^{\mathbb{P}}, \quad \text{(A.29)}$$

where the second equality in (A.28) substitutes the variance-matching identity $(\Gamma_1^j)^2 + (\Gamma_2^j)^2 = \gamma^j - \kappa^j - 2\delta^j$ from (A.18), and the third uses $\xi^j \equiv \kappa^j/\gamma^j$ together with $\gamma^j z_t^j = 2\,\mathcal{L}_t(M_{t,t+1}^j)$; in (A.29) the third equality uses $\sqrt{\gamma^{\mathbb{P}}(\gamma^{\mathbb{P}} - 2\delta^{\mathbb{P}} - \lambda^{\mathbb{P}})} = \gamma^{\mathbb{P}}(1-\xi^{\mathbb{P}}) - \delta^{\mathbb{P}}$, and the fourth uses $\gamma^{\mathbb{P}} z_t^{\mathbb{P}} = 2\,\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}})$.

Next, conjecture that $\Phi_t^{H(\infty)}$ is a homogeneous linear function of z_t and z_t^*:

$$\Phi_t^{H(\infty)} = c^{\mathbb{P}}\,\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}}) - c^{\mathbb{P}*}\,\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}*}). \quad \text{(A.30)}$$

Substituting (A.28)–(A.30) into (A.26)–(A.27) and collecting coefficients on $\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}})$ and $\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}*})$ (the home-factor $\delta^{\mathbb{P}} z_t^{\mathbb{P}}$ contributions from (A.28) and (A.29) cancel since both involve only m; no analogous cancellation occurs for the foreign factor):

$$\big[\mathbb{E}_t rx_{t+1}^{CT,(\infty)}\big]_{\mathcal{L}_t(M^{\mathbb{P}})} = \underbrace{1 + (1-\xi^{\mathbb{P}}) - 2(1-\xi^{\mathbb{P}})}_{=\,\xi^{\mathbb{P}}}\;\underbrace{-\tfrac{2\delta^{\mathbb{P}}}{\gamma^{\mathbb{P}}} + \tfrac{2\delta^{\mathbb{P}}}{\gamma^{\mathbb{P}}}}_{=\,0} - c^{\mathbb{P}} \;\equiv\; \tilde{\xi}, \quad \text{(A.31)}$$

$$\big[\mathbb{E}_t rx_{t+1}^{CT,(\infty)}\big]_{\mathcal{L}_t(M^{\mathbb{P}*})} = -1 + (1-\xi^{\mathbb{P}*}) - \tfrac{2\delta^{\mathbb{P}*}}{\gamma^{\mathbb{P}*}} + c^{\mathbb{P}*} \;\equiv\; -\tilde{\xi}^*. \quad \text{(A.32)}$$

Putting terms together yields the result:

$$\mathbb{E}_t\big[rx_{t+1}^{CT,(\infty)}\big] = \tilde{\xi}\,\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}}) - \tilde{\xi}^*\,\mathcal{L}_t(M_{t,t+1}^{\mathbb{P}*}) + \mathbb{E}_t\big[\theta_{t,t+1}^{F,H(\infty)} - \theta_{t,t+1}^{F,F(\infty)}\big], \quad \text{(A.33)}$$

with $\tilde{\xi} = \xi^{\mathbb{P}} - c^{\mathbb{P}}$ and $\tilde{\xi}^* = \xi^{\mathbb{P}*} - c^{\mathbb{P}*} + 2\delta^{\mathbb{P}*}/\gamma^{\mathbb{P}*}$. Under symmetry $\theta_t^{F,H(1)} - \theta_t^{H,H(1)} = \theta_t^{F,F(1)} - \theta_t^{H,F(1)}$, $\delta^j = 0$, so $\tilde{\xi}^*$ collapses to $\xi^{\mathbb{P}*} - c^{\mathbb{P}*}$.

The affine recursion. To complete the proof, we derive $c^{\mathbb{P}}, c^{\mathbb{P}*}$. At $k = 1$ the cross-currency

Euler gives $\theta_t^{F,H(1)} - \theta_t^{H,H(1)} = \mathbb{E}_t\eta_{t+1} - \frac{1}{2}\mathrm{Var}_t\eta_{t+1} + \mathrm{Cov}_t(m_{t,t+1}, \eta_{t+1})$.Rearranging the holding-period decomposition of Lemma 3 (eq. (21)),

$$\theta_t^{F,H(k)} - \theta_t^{H,H(k)} = \big(\theta_t^{F,H(1)} - \theta_t^{H,H(1)}\big) + \Phi_t^{H(k)} + \mathbb{E}_t\big[\theta_{t+1}^{F,H(k-1)} - \theta_{t+1}^{H,H(k-1)}\big]. \tag{A.34}$$

On the RHS the first and third terms are linear, since $\mathbb{E}_t z_{t+1}^j = (1-\phi^j)\bar{z}^j + \phi^j z_t^j$; it remains to show $\Phi_t^{H(k)}$ is linear.

The convenience-yield basis follows from dividing the k-period home and foreign-on-home Eulers (5) and (8), so $\theta_t^{F,H(k)} - \theta_t^{H,H(k)} = \log \mathbb{E}_t[M_{t,t+k}] - \log \mathbb{E}_t[M_{t,t+k}e^{-\eta^{(k)}}]$, where $\eta^{(k)} = \sum_{\tau=1}^{k} \eta_{t+\tau}$. We guess and verify that the k-period valuation is exponential-affine in the states:

$$\mathbb{E}_t\big[M_{t,t+k}\, e^{-\eta^{(k)}}\big] = \exp\Big(- \mathcal{A}_k - \tfrac{1}{2}\gamma^{\mathbb{P}}\mathcal{B}_k^{\mathbb{P}} z_t^{\mathbb{P}} - \tfrac{1}{2}\gamma^{\mathbb{P}*}\mathcal{B}_k^{\mathbb{P}*} z_t^{\mathbb{P}*}\Big). \tag{A.35}$$

The model equations (24)-(26), repeated for convenience, and the $\eta-$process are:

$$m_{t,t+1} = -\tfrac{1}{2}\gamma^{\mathbb{P}} z_t^{\mathbb{P}} - \sqrt{\gamma^{\mathbb{P}} z_t^{\mathbb{P}}}\, u_{t+1}^{\mathbb{P}}, \quad \eta_{t+1} = \sum_j \big[\psi^j z_t^j + \Gamma_1^j \sqrt{z_t^j}\, u_{t+1}^j + \Gamma_2^j \sqrt{z_t^j}\, \epsilon_{t+1}^j\big], \tag{A.36}$$

$$z_{t+1}^j = (1-\phi^j)\bar{z}^j + \phi^j z_t^j - \sigma^j \sqrt{z_t^j}\, u_{t+1}^j. \tag{A.37}$$

Factor $M_{t,t+k} = M_{t,t+1}M_{t+1,t+k}$ and $\eta^{(k)} = \eta_{t+1} + \eta_{t+2}^{(k-1)}$, $\eta_{t+2}^{(k-1)} = \sum_{\tau=2}^{k} \eta_{t+\tau}$; by the law of iterated expectations,

$$\mathbb{E}_t\big[M_{t,t+k}e^{-\eta_{t+1}^{(k)}}\big] = \mathbb{E}_t\Big[M_{t,t+1}e^{-\eta_{t+1}}\, \mathbb{E}_{t+1}\big[M_{t+1,t+k}e^{-\eta_{t+2}^{(k-1)}}\big]\Big] \equiv \mathbb{E}_t e^{X_k}. \tag{A.38}$$

Using (A.35) at $k-1$ for the inner expectation of (A.38), and (A.37) for z_{t+1}^j,

$$\begin{aligned} X_k &= m_{t,t+1} - \eta_{t+1} - \mathcal{A}_{k-1} - \tfrac{1}{2}\gamma^{\mathbb{P}}\mathcal{B}_{k-1}^{\mathbb{P}} z_{t+1}^{\mathbb{P}} - \tfrac{1}{2}\gamma^{\mathbb{P}*}\mathcal{B}_{k-1}^{\mathbb{P}*} z_{t+1}^{\mathbb{P}*} \\ &= m_{t,t+1} - \eta_{t+1} - \mathcal{A}_{k-1} - \sum_{j\in\{\mathbb{P},\mathbb{P}*\}} \tfrac{1}{2}\gamma^j \mathcal{B}_{k-1}^j \big[(1-\phi^j)\bar{z}^j + \phi^j z_t^j - \sigma^j\sqrt{z_t^j}\, u_{t+1}^j\big]. \end{aligned} \tag{A.39}$$

X_k is affine in z_t, z_t^* and the four innovations $u_{t+1}^{\mathbb{P}}, u_{t+1}^{\mathbb{P}*}, \epsilon_{t+1}^{\mathbb{P}}, \epsilon_{t+1}^{\mathbb{P}*}$. Define $\beta_k^{\mathbb{P}}$ as the coefficient on $\sqrt{z_t^{\mathbb{P}}}\, u_{t+1}^{\mathbb{P}}$. Three terms of X_k which contain $u_{t+1}^{\mathbb{P}}$ are as follows:

$$m_{t,t+1} \to -\sqrt{\gamma^{\mathbb{P}}}\sqrt{z_t^{\mathbb{P}}}\, u_{t+1}^{\mathbb{P}}, \quad -\eta_{t+1} \to -\Gamma_1^{\mathbb{P}}\sqrt{z_t^{\mathbb{P}}}\, u_{t+1}^{\mathbb{P}}, \quad -\tfrac{1}{2}\gamma^{\mathbb{P}}\mathcal{B}_{k-1}^{\mathbb{P}} z_{t+1}^{\mathbb{P}} \to +\tfrac{1}{2}\gamma^{\mathbb{P}}\sigma^{\mathbb{P}}\mathcal{B}_{k-1}^{\mathbb{P}}\sqrt{z_t^{\mathbb{P}}}\, u_{t+1}^{\mathbb{P}}. \tag{A.40}$$

Summing the three,

$$\beta_k^{\mathbb{P}} = -\sqrt{\gamma^{\mathbb{P}}} - \Gamma_1^{\mathbb{P}} + \tfrac{1}{2}\gamma^{\mathbb{P}}\sigma^{\mathbb{P}}\mathcal{B}_{k-1}^{\mathbb{P}} = -\sqrt{\gamma^{\mathbb{P}}} + \sqrt{\gamma^{\mathbb{P}} - 2\delta^{\mathbb{P}} - \lambda^{\mathbb{P}}} + \tfrac{1}{2}\gamma^{\mathbb{P}}\sigma^{\mathbb{P}}\mathcal{B}_{k-1}^{\mathbb{P}}, \tag{A.41}$$

the last step using $\Gamma_1^{\mathbb{P}} = -\sqrt{\gamma^{\mathbb{P}} - 2\delta^{\mathbb{P}} - \lambda^{\mathbb{P}}}$. The four innovations are independent, and the ϵ_{t+1}^{j}-coefficient of X_k is $-\Gamma_2^{j}\sqrt{z_t^{j}}$, so $\mathrm{Var}_t X_k = \sum_{j\in\{\mathbb{P},\mathbb{P}*\}}\left[(\beta_k^{j})^2 + (\Gamma_2^{j})^2\right] z_t^{j}$.

Since $m = m^{\mathbb{P}}$, $\mathbb{E}_t[M_{t,t+k}] = 1$, so the basis $\theta_t^{F,H(k)} - \theta_t^{H,H(k)} = -\log \mathbb{E}_t[M_{t,t+k}e^{-\eta^{(k)}}]$ is affine and, by (A.34), $\Phi_t^{H(k)}$ is linear. Equating the $z_t^{\mathbb{P}}$-coefficient of $\log \mathbb{E}_t e^{X_k}$ with $-\frac{1}{2}\gamma^{\mathbb{P}}\mathcal{B}_k^{\mathbb{P}}$, and using $(\Gamma_2^{\mathbb{P}})^2 = \lambda^{\mathbb{P}} - \kappa^{\mathbb{P}}$,

$$\mathcal{B}_k^{\mathbb{P}} = 2(\pi_{\mathbb{P}}^{F,H} - \pi_{\mathbb{P}}^{H,H})/\gamma^{\mathbb{P}} + \left[\phi^{\mathbb{P}} + (\xi^{\mathbb{P}} + \delta^{\mathbb{P}}/\gamma^{\mathbb{P}})\sqrt{\gamma^{\mathbb{P}}}\,\sigma^{\mathbb{P}}\right]\mathcal{B}_{k-1}^{\mathbb{P}} - \tfrac{1}{4}\gamma^{\mathbb{P}}(\sigma^{\mathbb{P}})^2\left(\mathcal{B}_{k-1}^{\mathbb{P}}\right)^2, \tag{A.42}$$

and its mirror for $\mathcal{B}_k^{\mathbb{P}*}$.[29]

Setting $\mathcal{B}_k^{\mathbb{P}} = \mathcal{B}_{k-1}^{\mathbb{P}} = \mathcal{B}_\infty^{\mathbb{P}}$ in (A.42) gives the quadratic

$$\tfrac{1}{4}\gamma^{\mathbb{P}}(\sigma^{\mathbb{P}})^2\left(\mathcal{B}_\infty^{\mathbb{P}}\right)^2 + \left[1 - \phi^{\mathbb{P}} - (\xi^{\mathbb{P}} + \delta^{\mathbb{P}}/\gamma^{\mathbb{P}})\sqrt{\gamma^{\mathbb{P}}}\,\sigma^{\mathbb{P}}\right]\mathcal{B}_\infty^{\mathbb{P}} - \tilde{\pi}^{\mathbb{P}} = 0, \tag{A.43}$$

$$\mathcal{B}_\infty^{\mathbb{P}} = \frac{-\left[1 - \phi^{\mathbb{P}} - (\xi^{\mathbb{P}} + \delta^{\mathbb{P}}/\gamma^{\mathbb{P}})\sqrt{\gamma^{\mathbb{P}}}\,\sigma^{\mathbb{P}}\right] + \sqrt{\left[1 - \phi^{\mathbb{P}} - (\xi^{\mathbb{P}} + \delta^{\mathbb{P}}/\gamma^{\mathbb{P}})\sqrt{\gamma^{\mathbb{P}}}\,\sigma^{\mathbb{P}}\right]^2 + \gamma^{\mathbb{P}}(\sigma^{\mathbb{P}})^2\,\tilde{\pi}^{\mathbb{P}}}}{\frac{1}{2}\gamma^{\mathbb{P}}(\sigma^{\mathbb{P}})^2}, \tag{A.44}$$

where we define $\tilde{\pi}^{\mathbb{P}} = 2(\pi_{\mathbb{P}}^{F,H} - \pi_{\mathbb{P}}^{H,H})/\gamma^{\mathbb{P}}$.

To extract $c^{\mathbb{P}}$, match the $z_t^{\mathbb{P}}$-loading of (A.34) at $k = \infty$. From the exponential-affine form (A.35), the LHS basis $\theta_t^{F,H(\infty)} - \theta_t^{H,H(\infty)}$ loads $\frac{1}{2}\gamma^{\mathbb{P}}\mathcal{B}_\infty^{\mathbb{P}}$ on $z_t^{\mathbb{P}}$. On the RHS: i) the one-period basis $\theta_t^{F,H(1)} - \theta_t^{H,H(1)}$ contributes $\frac{1}{2}\gamma^{\mathbb{P}}\tilde{\pi}^{\mathbb{P}}$ (using $\tilde{\pi}^{\mathbb{P}} \equiv \mathcal{B}_1^{\mathbb{P}}$); ii) the convenience term premium $\Phi_t^{H(\infty)} = c^{\mathbb{P}}\mathcal{L}_t(M^{\mathbb{P}}) - c^{\mathbb{P}*}\mathcal{L}_t(M^{\mathbb{P}*})$ contributes $\frac{1}{2}\gamma^{\mathbb{P}}c^{\mathbb{P}}$; iii) the expected continuation $\mathbb{E}_t[\theta_{t+1}^{F,H(\infty)} - \theta_{t+1}^{H,H(\infty)}]$ contributes $\frac{1}{2}\gamma^{\mathbb{P}}\phi^{\mathbb{P}}\mathcal{B}_\infty^{\mathbb{P}}$, since $\mathbb{E}_t z_{t+1}^{\mathbb{P}} = (1-\phi^{\mathbb{P}})\bar{z}^{\mathbb{P}} + \phi^{\mathbb{P}} z_t^{\mathbb{P}}$. Equating and

[29] Write (A.42) as $\mathcal{B}_k^{\mathbb{P}} = f(\mathcal{B}_{k-1}^{\mathbb{P}})$ with $f(b) = \tilde{\pi}^{\mathbb{P}} + \left[\phi^{\mathbb{P}} + (\xi^{\mathbb{P}} + \delta^{\mathbb{P}}/\gamma^{\mathbb{P}})\sqrt{\gamma^{\mathbb{P}}}\,\sigma^{\mathbb{P}}\right] b - \frac{1}{4}\gamma^{\mathbb{P}}(\sigma^{\mathbb{P}})^2 b^2$, and define $\mathcal{B}_\infty^{\mathbb{P}}$ as the limit of the iteration initialized at $\mathcal{B}_1^{\mathbb{P}} = \tilde{\pi}^{\mathbb{P}}$. The fixed-point equation $f(b) = b$ has discriminant $(1-a)^2 + \gamma^{\mathbb{P}}(\sigma^{\mathbb{P}})^2\,\tilde{\pi}^{\mathbb{P}}$ with $a \equiv \phi^{\mathbb{P}} + (\xi^{\mathbb{P}} + \delta^{\mathbb{P}}/\gamma^{\mathbb{P}})\sqrt{\gamma^{\mathbb{P}}}\,\sigma^{\mathbb{P}}$, so we maintain the parameter restriction $\tilde{\pi}^{\mathbb{P}} \geq -(1-a)^2/[\gamma^{\mathbb{P}}(\sigma^{\mathbb{P}})^2]$, to ensure real fixed points exist, and assume parameter bounds such that the iteration converges.

dividing by $\frac{1}{2}\gamma^{\mathbb{P}}$ yields $\mathcal{B}^{\mathbb{P}}_{\infty} = \tilde{\pi}^{\mathbb{P}} + c^{\mathbb{P}} + \phi^{\mathbb{P}}\mathcal{B}^{\mathbb{P}}_{\infty}$, i.e.,

$$c^{\mathbb{P}} = (1-\phi^{\mathbb{P}})\,\mathcal{B}^{\mathbb{P}}_{\infty} - \tilde{\pi}^{\mathbb{P}}. \tag{A.45}$$

The Foreign loading follows identically, with $(\xi^{\mathbb{P}} + \delta^{\mathbb{P}}/\gamma^{\mathbb{P}})$ replaced by $(1-\xi^{\mathbb{P}*} - \delta^{\mathbb{P}*}/\gamma^{\mathbb{P}*})$: $c^{\mathbb{P}*} = \tilde{\pi}^{\mathbb{P}*} - (1-\phi^{\mathbb{P}*})\,\mathcal{B}^{\mathbb{P}*}_{\infty}$, where $\tilde{\pi}^{\mathbb{P}*} = 2(\pi^{F,H}_{\mathbb{P}*} - \pi^{H,H}_{\mathbb{P}*})/\gamma^{\mathbb{P}*}$, with $\mathcal{B}^{\mathbb{P}*}_{\infty}$ the stable root of

$$\tfrac{1}{4}\gamma^{\mathbb{P}*}(\sigma^{\mathbb{P}*})^2\big(\mathcal{B}^{\mathbb{P}*}_{\infty}\big)^2 + \big[1-\phi^{\mathbb{P}*} - (1-\xi^{\mathbb{P}*} - \delta^{\mathbb{P}*}/\gamma^{\mathbb{P}*})\sqrt{\gamma^{\mathbb{P}*}}\,\sigma^{\mathbb{P}*}\big]\mathcal{B}^{\mathbb{P}*}_{\infty} - \tilde{\pi}^{\mathbb{P}*} = 0. \tag{A.46}$$

□

A.9 Proposition 2 across the model variants

For every variant, Proposition 2 holds, but models differ through Ω_t in (A.33). The incomplete markets wedge follows (A.15) as in the previous section.

Conjecture that $\Phi^{H(\infty)}_t$ is affine in the states of the variant,

$$\Phi^{H(\infty)}_t = \sum_{j\in J_H} c^j\,\mathcal{L}_t(M^j) - \sum_{j\in J_F} c^j\,\mathcal{L}_t(M^j) + \Phi^{\theta}_t, \tag{A.47}$$

with Φ^{θ}_t being the component affine in the orthogonal convenience-demand states $(z^{\theta(*)}_t)$. The loadings c^j can be solved following the same steps as Proposition 2′. Substitute (A.16)–(A.47) into Proposition 2 and collect the coefficient of each $\mathcal{L}_t(M^j)$. A Home factor receives $-c^j$ from $-\Phi^{H(\infty)}_t$, $+(1-\xi^j)$ and $-2\delta^j/\gamma^j$ from $\mathcal{L}_t(e^{-\eta})$, and $-2(1-\xi^j)$ and $+2\delta^j/\gamma^j$ from $\mathcal{C}_t(M, e^{-\eta})$ — the two δ^j contributions cancel. A Foreign factor receives $+c^j$ from $-\Phi^{H(\infty)}_t$, $+(1-\xi^j)$ and $-2\delta^j/\gamma^j$ from $\mathcal{L}_t(e^{-\eta})$, with nothing from the coentropy: the δ^j does not cancel.

The total Home loading is then $\tilde{\xi}^j$ and the net Foreign loading is $-\tilde{\xi}^j$, where, depending on each model:

$$\tilde{\xi}^j = \begin{cases} \xi^{\mathbb{P}} - c^{\mathbb{P}} & j = \mathbb{P}, \\ \xi^{\mathbb{P}*} - c^{\mathbb{P}*} + 2\delta^{\mathbb{P}*}/\gamma^{\mathbb{P}*} & j = \mathbb{P}*, \\ \xi^{\mathbb{T}} - c^{\mathbb{T}} - 1 & j = \mathbb{T}, \\ \xi^{\mathbb{T}*} - c^{\mathbb{T}*} - 1 & j = \mathbb{T}*. \end{cases} \tag{A.48}$$

The proposition is written as:

$$\mathbb{E}_t\big[rx_{t+1}^{CT,(\infty)}\big] = \sum_{j\in J_H} \tilde{\xi}^j\, \mathcal{L}_t(M^j) - \sum_{j\in J_F} \tilde{\xi}^j\, \mathcal{L}_t(M^j) + \mathbb{E}_t\big[\Delta\theta_t^{(\infty)}\big] - \Phi_t^{\theta}. \tag{A.49}$$

Baseline. $m = m^{\mathbb{P}}$, $m^* = m^{\mathbb{P}*}$; $J_H = \{\mathbb{P}\}$, $J_F = \{\mathbb{P}*\}$, $\Phi_t^{\theta} = 0$:

$$\mathbb{E}_t\big[rx_{t+1}^{CT,(\infty)}\big] = \tilde{\xi}^{\mathbb{P}}\, \mathcal{L}_t(M^{\mathbb{P}}) - \tilde{\xi}^{\mathbb{P}*}\, \mathcal{L}_t(M^{\mathbb{P}*}) + \mathbb{E}_t\big[\Delta\theta_t^{(\infty)}\big], \tag{A.50}$$

with $\tilde{\xi}^{\mathbb{P}} = \xi^{\mathbb{P}} - c^{\mathbb{P}}$ and $\tilde{\xi}^{\mathbb{P}*} = \xi^{\mathbb{P}*} - c^{\mathbb{P}*} + 2\delta^{\mathbb{P}*}/\gamma^{\mathbb{P}*}$, recovering Proposition 2′ when $\delta^{\mathbb{P}(*)} = 0$.

Case 1 — with transitory innovations. $m = m^{\mathbb{P}} + m^{\mathbb{T}}$, $m^* = m^{\mathbb{P}*} + m^{\mathbb{T}*}$; $J_H = \{\mathbb{P}, \mathbb{T}\}$, $J_F = \{\mathbb{P}*, \mathbb{T}*\}$. $\Phi_t^{H(\infty)}$ is conjectured affine on all four factors. Since convenience yields in (25) load only on permanent factors, $\delta^{\mathbb{T}} = \delta^{\mathbb{T}*} = 0$, so:

$$\mathbb{E}_t\big[rx_{t+1}^{CT,(\infty)}\big] = \tilde{\xi}^{\mathbb{P}}\mathcal{L}_t(M^{\mathbb{P}}) + \tilde{\xi}^{\mathbb{T}}\mathcal{L}_t(M^{\mathbb{T}}) - \tilde{\xi}^{\mathbb{P}*}\mathcal{L}_t(M^{\mathbb{P}*}) - \tilde{\xi}^{\mathbb{T}*}\mathcal{L}_t(M^{\mathbb{T}*}) + \mathbb{E}_t\big[\Delta\theta_t^{(\infty)}\big], \tag{A.51}$$

with $\tilde{\xi}^{\mathbb{P}} = \xi^{\mathbb{P}} - c^{\mathbb{P}}$, $\tilde{\xi}^{\mathbb{P}*} = \xi^{\mathbb{P}*} - c^{\mathbb{P}*} + 2\delta^{\mathbb{P}*}/\gamma^{\mathbb{P}*}$, $\tilde{\xi}^{\mathbb{T}} = \xi^{\mathbb{T}} - c^{\mathbb{T}} - 1$, and $\tilde{\xi}^{\mathbb{T}*} = \xi^{\mathbb{T}*} - c^{\mathbb{T}*} - 1$. If convenience yields also load on transitory factors ($\delta^{\mathbb{T}*} \neq 0$), the foreign transitory loading picks up the analogous asymmetry correction $\tilde{\xi}^{\mathbb{T}*} = \xi^{\mathbb{T}*} - c^{\mathbb{T}*} - 1 + 2\delta^{\mathbb{T}*}/\gamma^{\mathbb{T}*}$; home loadings are unchanged, but the model would now predict a non-trivial structural loading on $\mathcal{L}_t(M^{\mathbb{T}*})$ in long-maturity CIP.

Case 2 — with additional orthogonal convenience states. $m = m^{\mathbb{P}}$; $J_H = \{\mathbb{P}\}$, $J_F = \{\mathbb{P}*\}$. The convenience yields load the risk factors and the orthogonal $z_t^{\theta(*)}$ states, so $\Phi_t^{H(\infty)}$ carries an additional term $\Phi_t^{\theta} \neq 0$. As long as these orthogonal states affect a given bond's convenience symmetrically for Home and Foreign investors:

$$\mathbb{E}_t\big[rx_{t+1}^{CT,(\infty)}\big] = \tilde{\xi}^{\mathbb{P}}\mathcal{L}_t(M^{\mathbb{P}}) - \tilde{\xi}^{\mathbb{P}*}\mathcal{L}_t(M^{\mathbb{P}*}) + \mathbb{E}_t\big[\Delta\theta_t^{(\infty)}\big] - \Phi_t^{\theta}, \tag{A.52}$$

with $\tilde{\xi}^{\mathbb{P}}$ and $\tilde{\xi}^{\mathbb{P}*}$ as in (A.50).

A.10 Asset Pricing Bounds in the Affine Framework

In this Appendix, we derive the market structure under which Lemma 2 holds with equality.

Let the log return of a generic risky portfolio in the economy be $r^p_{t,t+1} = \mathbb{E}_t[r^p_{t,t+1}] + \beta^p_t u^{\mathbb{P}}_{t+1}$, where β^p_t represents the portfolio's chosen exposure to the single domestic macroeconomic shock (and analogously in the Foreign country). From Assumption 2, the Euler equation for any risky asset providing no convenience yield is $\mathbb{E}_t[M_{t,t+1}R^p_{t,t+1}] = 1$. We substitute the baseline affine SDF $m_{t,t+1} = -\frac{1}{2}\gamma^{\mathbb{P}} z^{\mathbb{P}}_t - \sqrt{\gamma^{\mathbb{P}} z^{\mathbb{P}}_t} u^{\mathbb{P}}_{t+1}$ and the generic risky asset return process into the Euler equation:

$$\mathbb{E}_t \left[\exp \left(-\frac{1}{2}\gamma^{\mathbb{P}} z^{\mathbb{P}}_t - \sqrt{\gamma^{\mathbb{P}} z^{\mathbb{P}}_t} u^{\mathbb{P}}_{t+1} + \mathbb{E}_t[r^p_{t,t+1}] + \beta^p_t u^{\mathbb{P}}_{t+1} \right) \right] = 1$$

Evaluating this expectation using the moment generating function of a standard normal distribution yields:

$$\exp \left(-\frac{1}{2}\gamma^{\mathbb{P}} z^{\mathbb{P}}_t + \mathbb{E}_t[r^p_{t,t+1}] + \frac{1}{2}(\beta^p_t - \sqrt{\gamma^{\mathbb{P}} z^{\mathbb{P}}_t})^2 \right) = 1$$

Taking logs of both sides and rearranging:

$$\mathbb{E}_t[r^p_{t,t+1}] = \frac{1}{2}\gamma^{\mathbb{P}} z^{\mathbb{P}}_t - \frac{1}{2}(\beta^p_t - \sqrt{\gamma^{\mathbb{P}} z^{\mathbb{P}}_t})^2.$$

Because investors can freely borrow or lend at the risk-free rate, they can use continuously rebalanced portfolios to achieve their desired risk exposure, β^p_t. To maximize the expected log return of their portfolio, the investor chooses the specific allocation where $\beta^p_t = \sqrt{\gamma^{\mathbb{P}} z^{\mathbb{P}}_t}$. Substituting this optimal portfolio weight (exposure) back into the expected return gives $\mathbb{E}_t[\hat{r}_{t,t+1}] = \frac{1}{2}\gamma^{\mathbb{P}} z^{\mathbb{P}}_t$. The realized log return of this optimized portfolio is therefore:

$$\hat{r}_{t,t+1} = \frac{1}{2}\gamma^{\mathbb{P}} z^{\mathbb{P}}_t + \sqrt{\gamma^{\mathbb{P}} z^{\mathbb{P}}_t} u^{\mathbb{P}}_{t+1}$$

Notice that this is exactly equal to $-m_{t,t+1}$.

Finally, we verify that this saturates Lemma 2. By definition in our affine setup, the conditional entropy of the pricing kernel is $\mathcal{L}_t(M_{t,t+1}) = \frac{1}{2}\gamma^{\mathbb{P}} z^{\mathbb{P}}_t$. The Home Euler equation for the risk-free bond is given by (5). Since $\mathbb{E}_t[M_{t,t+1}] = 1$ in our baseline specification, the log risk-free rate is exactly $\log R_t = -\theta^{H,H(1)}_t$. We evaluate the right-hand side of the Lemma 2 inequality using the expected log return of our optimal portfolio and the log risk-free rate:

$$\mathbb{E}_t\left[\log \frac{\hat{R}_{t,t+1}}{R_t}\right] - \theta_t^{H,H(1)} = \mathbb{E}_t[\hat{r}_{t,t+1}] - \log R_t - \theta_t^{H,H(1)}$$

$$= \frac{1}{2}\gamma^{\mathbb{P}} z_t^{\mathbb{P}} - (-\theta_t^{H,H(1)}) - \theta_t^{H,H(1)} = \frac{1}{2}\gamma^{\mathbb{P}} z_t^{\mathbb{P}},$$

so Lemma 2 holds with equality.

In our baseline, we proxy $\hat{r}$ with the aggregate stock index; however, we have verified that our main results are robust to directly constructing the growth-optimal leveraged portfolio following Bansal and Lehmann (1997).

A.11 Holding-Period Convenience in the Affine Model

This appendix derives the relationship between the one-period holding period convenience yield on an infinite maturity bond, and the convenience yield to maturity on the same bond. Because the baseline pricing kernel is assumed to be a pure martingale ($m = m^{\mathbb{P}}$), $\mathbb{E}_t[M_{t,t+k}] = 1$, and so $\theta_t^{F,H(k)} - \theta_t^{H,H(k)} = -\log \mathbb{E}_t[M_{t,t+k}\, e^{-\eta^{(k)}}]$ is affine in the state variables as derived in (A.35). The CIP deviation $\theta_t^{F,H(k)} - \theta_t^{F,F(k)}$ likewise admits an affine representation. Accordingly, write

$$\theta_t^{F,H(k)} - \theta_t^{F,F(k)} \;=\; A_k^{CIP} \;+\; B_k^{CIP,\mathbb{P}}\, z_t^{\mathbb{P}} \;+\; B_k^{CIP,\mathbb{P}*}\, z_t^{\mathbb{P}*}, \tag{A.53}$$

where A_k^{CIP} and $B_k^{CIP,j}$ are maturity-k coefficients on the CIP deviation. We then divide $(1-\omega^{(k)})(\theta_t^{F,H(k)} - \theta_t^{F,F(k)}) = c_k + \mathbb{E}_t[\theta_{t+1}^{F,H(k-1)} - \theta_{t+1}^{F,F(k-1)}]$ through by $\theta_t^{F,H(k)} - \theta_t^{F,F(k)}$, where c_k is the maturity-k mean. Substituting (A.53) and the state-variable laws of motion $\mathbb{E}_t\, z_{t+1}^j = (1-\phi^j)\bar{z}^j + \phi^j\, z_t^j$, and rearranging, yields:

$$\omega^{(k)} \;=\; 1 \;-\; \frac{c_k + A_{k-1}^{CIP} + B_{k-1}^{CIP,\mathbb{P}}(1-\phi^{\mathbb{P}})\bar{z}^{\mathbb{P}} + B_{k-1}^{CIP,\mathbb{P}*}(1-\phi^{\mathbb{P}*})\bar{z}^{\mathbb{P}*} + \phi^{\mathbb{P}}\, B_{k-1}^{CIP,\mathbb{P}}\, z_t^{\mathbb{P}} + \phi^{\mathbb{P}*}\, B_{k-1}^{CIP,\mathbb{P}*}\, z_t^{\mathbb{P}*}}{A_k^{CIP} + B_k^{CIP,\mathbb{P}}\, z_t^{\mathbb{P}} + B_k^{CIP,\mathbb{P}*}\, z_t^{\mathbb{P}*}}. \tag{A.54}$$

Consider the infinite-maturity limit $k \to \infty$: $c_k \to c_\infty$, $A_k^{CIP} \to A_\infty^{CIP}$, and $B_k^{CIP,j} \to B_\infty^{CIP,j}$ for $j \in \{\mathbb{P}, \mathbb{P}*\}$. Assuming symmetric persistence $\phi^{\mathbb{P}} = \phi^{\mathbb{P}*} \equiv \phi$, matching the coefficient on each z_t^j in (A.54) gives $(1-\omega^{(\infty)})\, B_\infty^{CIP,j} = \phi\, B_\infty^{CIP,j}$, so

$$\omega^{(\infty)} \;=\; 1 - \phi, \tag{A.55}$$

which is constant. The result depends only on the AR(1) structure of the state variables; the specific values of A_k^{CIP} and $B_k^{CIP,j}$ in (A.53) do not appear in the limiting expression.

B Data and Empirics

B.1 Data Sources

CIP Deviations. We use CIP deviations at 3-month, 1-year and 10-year maturities from Du, Im, and Schreger (2018a) for 6 industrialized countries relative to the U.S.: Australia, Canada, euro area, Japan, Switzerland and U.K. The sample begins 1997:01 and ends 2021:03, although the panel is unbalanced as convenience yields are not available from the start of the sample in all jurisdictions. Owing to the availability of swap-spread data used to measure within-country convenience yields, as discussed below, our sample begins in 2000:01. We use the 3-month and 1-year convenience yields to construct 6-month convenience yields by linearly interpolating according to $CIP_t^{(6M)} = \frac{2}{3}CIP_t^{(3M)} + \frac{1}{3}CIP_t^{(1Y)}$. Table B.1 summarizes the start dates of the convenience yields. The data runs through 2021:03, which defines the end-point of our sample.

Table B.1: Du, Im, and Schreger (2018a) CIP Data Start Dates

Country	6-month Start Date	10-year Start Date
Australia	1997:10	1997:03
Canada	2001:02	2000:02
Euro Area	1999:01	1999:01
Japan	1997:06	1997:06
Switzerland	1998:09	1998:09
U.K.	1997:01	1997:01

Notes: The 6-month start date is the $\max\{\text{3-month}, \text{1-year}\}$ start date.

Bond Yields. We use nominal zero-coupon government bond yields at 6-month and 10-year maturities for the AE economies. Table B.2 summarizes the sources of nominal zero-coupon government bond yields for the economies in our study.

Table B.2: Yield Curve Data Sources

Country	Sources
U.S.	Gürkaynak, Sack, and Wright (2007)
Australia	Reserve Bank of Australia
Canada	Bank of Canada
Euro Area	Bundesbank (German Yields)
Japan	Wright (2011) and Bank of England
Switzerland	Swiss National Bank
U.K.	Anderson and Sleath (2001)

Exchange Rates. Exchange rate data is from *Datastream*, reflecting end-of-month spot rates *vis-à-vis* the U.S. dollar.

Swap Rates and Extensions. We use interest-rate swap rate data for two jurisdictions—the U.S. and the E.A.—and for two maturities—6 months and 10 years. The swap market only became sufficiently developed and liquid for the other G.7 currency areas around 2007. Thus, we do not consider swap rates for other G.7 currencies.

We begin by collecting the following data on interest-rate swap rates from *Datastream*. First, at the 6-month maturity, we collect overnight indexed swap (OIS) rates. The start date for these series is 2001:10 for the U.S. and 1999:10 for the euro area and they run until the end of our sample. These OIS swap rates are indexed to the effective Fed Funds rate and EONIA for the U.S. and euro area, respectively. Second, for the U.S. at the 10-year maturity, we start with interest-rate swaps rates that are indexed to LIBOR. This series becomes available in June 2003 and runs until the end of our sample. Third, for the euro area and at a 9-year maturity—which we use as a proxy for the 10-year due to data availability—we again use OIS rates, which become available in August 2005.

For the U.S. case, we then extend the 6-month and 10-year interest-rate swap rate series by back-filling them with 6-month and 10-year risk- and convenience-free AAA corporate bond rates, respectively, from the FRED database, à la Krishnamurthy and Vissing-Jorgensen (2012).[30] The correlations between the interest-rate swap and AAA corporate series are 89% and 82% for the 6-month and 10-year maturities, respectively. After accounting for a modest

[30] The codes are HQMCB6MT and HQMCB10YRP for the 6-month and 10-year maturities, respectively.

level-effect (the variances between the series are similar), our extended U.S. interest-rate swap series closely tracks that from Du et al. (2023), who use data from JP Morgan Markets.[31] Overall, we use our extended U.S. interest-rate swap series beginning in 1997:01, to match the start date of the CIP deviations from Du et al. (2018a), although, in practice, data beginning only in 1999:10 is used due to availability of euro area data (see below).

For the euro area case, we extend the 9-year OIS rates by back-filling them with the 6-month OIS rates, since these display a correlation of 91%. Again, we account for a level effect (the variances are again comparable). In all, this allows us to extend our long-maturity series from a start date of 2005:08 to one of 1999:10.

Equities. We use three main sources related to equities. First, we obtain equity price indices of large and mid-cap sized firms for the U.S., Australia, Canada, Euro Area, Japan, Switzerland and the U.K. from *MSCI.* These equity returns are used to construct representative *ex post* measures of aggregate risk in each jurisdiction. Second, we collect data on dividend-price ratios and equity prices for each G.7 currency area from *Global Financial Data.* Specifically, we collect these data for the S&P-500 (U.S.), EuroStoxx-50 (E.A.), FTSE-100 (U.K.), TOPIX (Japan), S&P/ASX-200 (Australia), S&P/TSX-300 (Canada) and SMI (Switzerland).[32] Third, we collect buyback yield data from *Bloomberg.* We use these data to construct *ex ante* measures of equity risk premia as described in the main text. As with our other data, we use the end-of-month observations for all equity-related series.

Expectations. We use survey data from *Consensus Economics* on future GDP growth, inflation and exchange rates.

VIX. To proxy physical entropy $\mathcal{L}\left(R^g_{t,t+1}/R_t\right) \approx \frac{T-t}{2} VIX_t^2$ (Martin (2017), result 3) for a 6-month holding period, we need the 6-month VIX. However, the 6-month VIX is available only from 2005:01 for the euro area (VSTOXX) and from 2008:02 for the U.S. (S&P500) and is unavailable for other AE jurisdictions. The standard 1-month VIX, which could be used as a proxy, is only available from the start of our sample for the U.S., with data for the majority of other AE jurisdictions available only after the global financial crisis. Therefore, we use the

[31] Presumably, as there was no OIS market in 1996, JP Morgan Markets performs a back-fill similar to us.
[32] This data is not available for the greater set of large- and mid-cap public firms in *MSCI.*

Gaussian approximation outlined in Martin (2017), result 4: $VIX_t^2 \approx \frac{1}{T-t}\text{var}_t(\log R^g_{t,t+1})$ (see Figure B.5a for details of this construction), such that $\mathcal{L}(R^g_{t,t+1}/R_t) \approx \frac{1}{2}\text{var}_t(\log R^g_{t,t+1})$. For the U.S., the correlation between $\frac{1}{2}VIX_t^2$ and $\frac{1}{2}\text{var}_t(\log R^g_{t,t+1})$ is 58%, with comparable magnitudes.

Government Debt-to-GDP. We source quarterly general government total and long-term nominal debt as a percentage of GDP for each AE economy from the *World Bank Quarterly Public Sector Debt Database*. We linearly interpolate from quarterly to monthly frequency. The control in Table 2 is the U.S. ratio minus the equally-weighted average of the six remaining AE ratios, matching the aggregation used for our other cross-country differentials.

Intermediary leverage. We measure intermediary leverage at a monthly frequency as the inverse of the intermediary capital ratio constructed by He et al. (2017).

B.2 Constructing Expected Equity Returns

In our baseline, we proxy for expected future dividend growth using a linear combination of the current dividend–price ratio, two semi-annual lags of past dividend growth, and next-year GDP growth expectations from *Consensus Economics*. Specifically, we regress six-month (one-period) future dividend growth g_t on the contemporaneous dividend–price ratio D_t/P_t, trailing six-month and twelve-month dividend growth g_{t-1} and g_{t-2}, and next year's GDP growth expectations $g_t^{GDP,e}$ from *Consensus Economics* country-by-country, and use the predicted value as our proxy for g_t^e:

$$g_t^e \;:=\; \hat{g}_t \;=\; \hat{\alpha} \;+\; \hat{\beta}_1\,(D_t/P_t) \;+\; \hat{\beta}_2\, g_{t-1} \;+\; \hat{\beta}_3\, g_{t-2} \;+\; \hat{\beta}^{GDP}\, g_t^{GDP,e}. \tag{B.1}$$

The results from estimating regression (B.1) for the U.S. are presented in Table B.3. The fit is strong — adjusted R^2 of 46% — with both lagged dividend growth at the one-period lag (g_{t-1}) and one-year-ahead GDP growth expectations ($g_t^{GDP,e}$) highly significant. Combining past dividend growth with forward-looking survey data on GDP allows our measure to capture both trends in risk and cyclical dynamics. We estimate analogous regressions for each country in our sample, and the resulting per-country $\hat{g}_t^e$ enters the ERP computation in equation (2).

Encouragingly, Figure B.1 shows that our measure of expected U.S. dividend growth constructed from B.1 co-moves strongly with analyst-based survey data from De La'O and Myers

Table B.3: U.S. (S&P 500) Future Dividend Growth Regressions

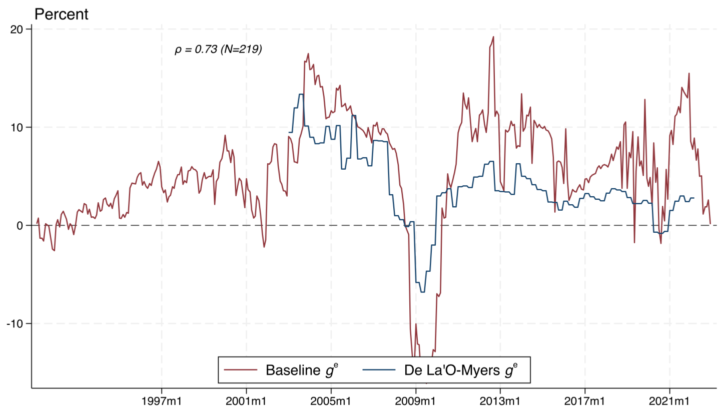
Panel A: U.S. (S&P 500) coefficient estimates	
Dep. Var.: g_t	
D/P_t	-3.05
	(4.34)
g_{t-1}	0.60^{***}
	(0.13)
g_{t-2}	0.01
	(0.13)
$g_t^{GDP,e}$	3.20^{***}
	(1.16)
Adjusted R^2	0.46
N	255

Panel B: per-country adjusted R^2							
	USD	AUD	CAD	CHF	EUR	GBP	JPY
Adjusted R^2	0.46	0.32	0.19	0.11	0.23	0.34	0.18

Notes: Coefficient estimates from regression (B.1) for the U.S. (S&P 500). Newey and West (1987) HAC standard errors with 6 lags reported in parentheses; ***, **, and * denote $p < 0.01$, $p < 0.05$, and $p < 0.1$, respectively. All regressions estimated over the headline sample 2000m1–2021m3 for every country.

(2021) on dividend growth expectations over the same horizon.

Figure B.1: Comparison of Baseline g^e Measure with De La'O and Myers (2021) Survey Data



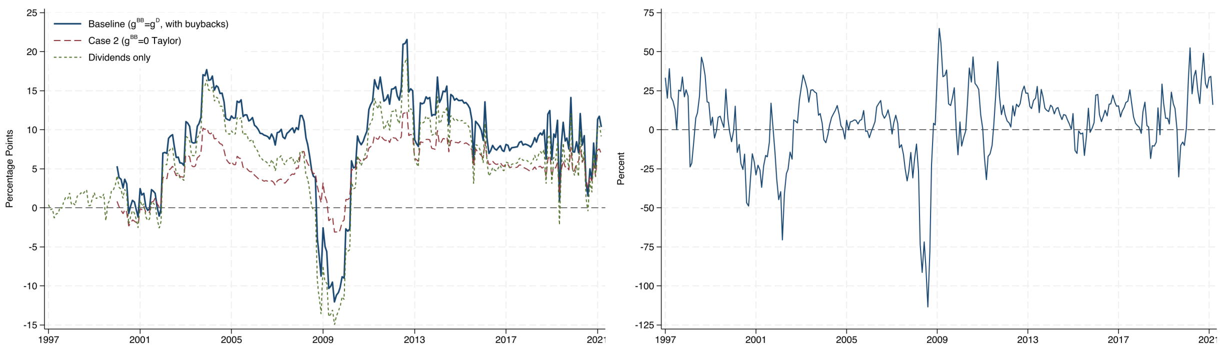
Notes: Figure B.1 presents time series of our baseline U.S. expected 6-month-ahead dividend growth (g^e) measure with analogous measure based on analyst survey data from De La'O and Myers (2021). The two series have a correlation of 0.73 beginning in January 2003, when the latter series becomes available.

Atkeson et al. (2025) argue that public firms' valuations since the 1970s increasingly reflect cash flow paid out as buybacks — implying that the total payout to price ratio may be significantly higher and more volatile than the dividend yield alone. For the U.S., the Euro Area, and the U.K., where firm-level net-repurchase data are available, we include the buyback yield B_t/P_t in the total cash-flow yield $TCF_t/P_t = D_t/P_t + B_t/P_t$ entering equation (2); for AUD, CAD, CHF, and JPY, where a comparable series is not available, we set $B_t/P_t \equiv 0$.

We consider three alternative specifications for equity premia construction. In our baseline we assume $g^e = g^d$, which implies that buybacks are expected to grow at the same pace as dividends. This is the outcome of a model where firms pursue a constant payout policy, in a world where buybacks are becoming more important. As robustness we consider the case where buyback growth is unexpected, such that $g^e_t = \frac{D_t}{D_t+BB_t} \times g^d_t + \frac{BB_t}{D_t+BB_t} \times 0$, consistent with a Taylor approximation of (2), and also a dividends-only alternative where buybacks are set to zero ($B_t = 0$). Figure B.2 contrasts expected U.S. equity premia obtained using equation (2) and measuring expected dividend growth using (B.1) (Panel a), contrasting the three cases outlined above. Panel b plots realized equity premia for comparison. Our headline regression results are robust to using any of them.

Figure B.2: U.S. Expected and Realized Equity Excess Returns

(a) U.S. Expected Equity Excess Returns (b) U.S. Realized Equity Excess Returns



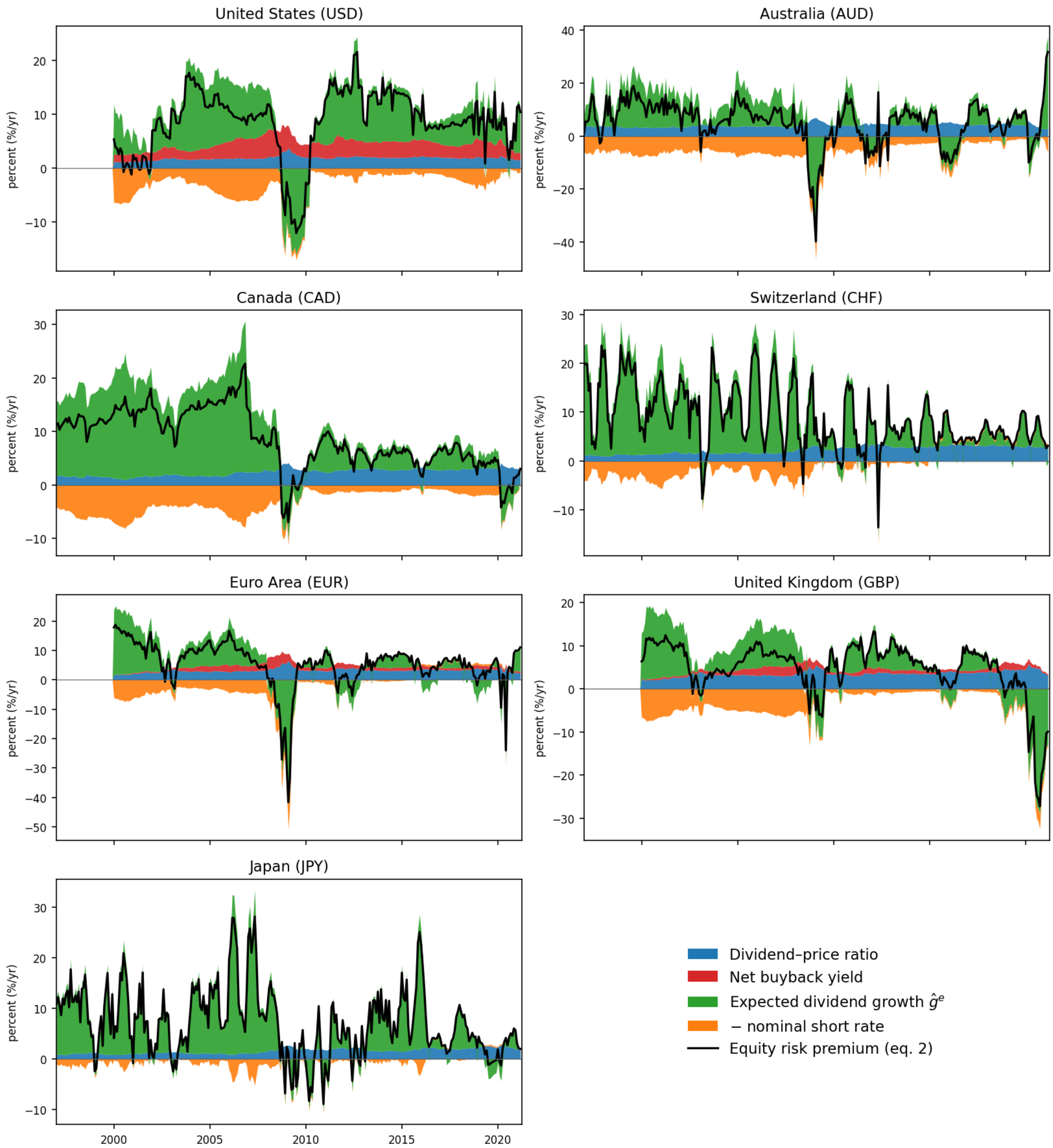
Note. Panels B.2a and B.2b display time series of our baseline measures of U.S. expected and realized 6-month equity excess returns, respectively. The expected equity excess return is constructed from the Gordon growth formula in equation (2), with expected dividend growth $\hat{g}^e_t$ taken from the predictive regression in equation (B.1). Panel B.2a overlays three constructions of the cash-flow yield: the headline baseline imposing $g^{BB} = g^D$ with the buyback yield included (solid navy); the Case 2 alternative imposing $g^{BB} = 0$ via a first-order Taylor approximation, so the growth term on the total payout is scaled by the dividend share $D/P/(D/P + B/P)$ (dashed maroon); and a dividends-only construction that drops the buyback yield entirely (dotted green). The sample runs from 1997:01 to 2021:03 to match data on CIP deviations.

Dissecting Expected Equity Risk Premia Figure B.3 offers an empirical decomposition of our estimate for *ex ante* U.S. equity premia in the four objects on the right-hand-side of (2). Across all seven countries, the dividend-growth forecast $\hat{g}^e_{z,t}$ accounts for the bulk of the high-frequency variation in the ERP, with synchronised global troughs in 2008–09 and 2020. The raw dividend–price ratio $D_{z,t}/P_{z,t}$ (blue) is small and structurally stable everywhere — in the 1–4% band — adding negligible cyclical content; cross-country differences in its level largely reflect long-run differences in payout policy and valuation. Declining nominal rates contribute consistently negative in the early sample for currencies with high pre-2008 short rates (USD, AUD, CAD, GBP), but shrinks toward zero — and turns positive for CHF and EUR, mechanically lifting the level of every country's ERP through the back half of the sample.

Figure B.4 repeats the exercise in relative terms, decomposing the U.S. net AE ERP differential into the same four components. The expected dividend-growth differential $\Delta\hat{g}_t$ again accounts for the bulk of the high-frequency variation, with the same global troughs in 2001–03, 2008–09, and 2020 but now compounded by a clear secular trend: U.S. $\hat{g}$ rises relative to the foreign average from 2010 onwards, and indeed the AE expected dividend growth falls over our sample. The buyback-yield differential $\Delta(B_t/P_t)$ is a near-constant level shifter of roughly 2 percentage points, reflecting that comparable net-repurchase data are available for the U.S. (and partly the Euro Area and the U.K.) but identically zero for AUD, CAD, CHF, and JPY. The raw dividend–price differential $\Delta(D_t/P_t)$ contributes essentially zero throughout — U.S. and foreign dividend yields track each other closely, adding no meaningful cyclical or trend content. Declining nominal-rate differentials $-\Delta r_t$ are also an important secular driver: the component is negative through the early 2000s when U.S. short rates exceeded the foreign average, and drops again after 2015 as CHF, EUR, and JPY policy rates fell to the zero lower bound while the Federal Reserve raised rates.

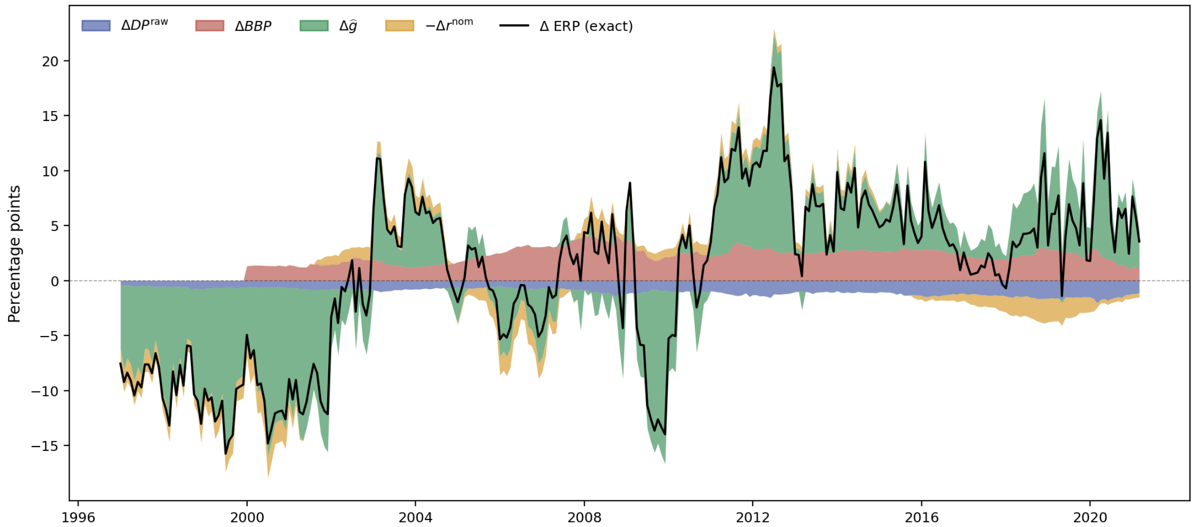
Estimating Jensen's adjustment. Figure B.5 compares our constructed conditional variance with the corresponding VIX measure. The advantage of our measure is that it can be constructed over the full sample for all of our AE countries. Within each month we compute the sample variance of *daily* log equity excess returns, scale it to an annual horizon by multiplying by the number of trading days—which requires only that daily excess returns be serially uncorrelated, the approximation underlying Martin (2017), Result 4—and divide by two. The resulting correction is expressed in percentage points per year, the same units as the expected equity premium from which it is subtracted in (28), and as $\frac{1}{2}\text{VIX}_t^2 = \text{VIX}_t^2/200$ with the VIX quoted in annualized percent. We do not expect the two series to coincide, but they should

Figure B.3: Decomposing AE Equity Premia: $\log \mathbb{E}_t\left[\frac{R^{eq}_{t,t+1}}{R_t}\right] \approx \frac{D_t}{P_t} + \frac{B_t}{P_t} + g^e_t - r_t$



Notes: The seven panels of Figure B.3 decompose our baseline measure of the country-level log equity risk premium $\log \mathbb{E}_t[R^{eq}_{z,t,t+1}/R_{z,t}]$ into its four contributing objects (2): the raw dividend–price ratio $D_{z,t}/P_{z,t}$ (blue), the net buyback yield $B_{z,t}/P_{z,t}$ (red; available only for the U.S., the Euro Area, and the U.K.; set to zero for AUD, CAD, CHF, and JPY), the expected dividend-growth forecast $\hat{g}_{z,t+6}$ (green), and minus the 6-month nominal short rate $-r_{z,t}$ (orange). The thick black line in each panel is the exact ERP from equation (2). Sample runs from 1997:01 to 2021:03.

Figure B.4: Decomposing the U.S.–minus-AE Equity Premium Differential:
$\log \mathbb{E}_t[R^{eq,US}/R^{US}] - \frac{1}{6}\sum_z \log \mathbb{E}_t[R^{eq,z}/R^z] \approx \Delta(D/P) + \Delta(B/P) + \Delta\hat{g} - \Delta r$



Notes: Stacked decomposition of the U.S. minus AE (equally-weighted, EW6) ERP differential into its contributing factors (2): the raw dividend–price ratio differential $\Delta D_t/P_t$ (blue), the net-buyback yield differential $\Delta(B_t/P_t)$ (red), the expected dividend-growth differential $\Delta\hat{g}_t$ (green), and minus the 6-month nominal short-rate differential $-\Delta r_t$ (orange). The thick black line is the constructed ΔERP. Sample 1997:01–2021:03; per-country dividend-growth forecasts estimated over the headline 2000:01–2021:03 window.

co-move: the systematic wedge between them can be interpreted as a variance risk premium.[33]

B.3 Estimating a Mapping from CIP to Cross-Country Convenience Yields

While we cannot do this for our long-maturity equilibrium, we estimate the coefficient χ_k^* in $\theta_t^{F,H(k)} - \theta_t^{F,F(k)} = \frac{1}{1-\chi_k^*} CIP_t^{(k)}$ following the approach of Jiang et al. (2021a). We focus on the 6-month CIP deviation, our measure of short-maturity convenience, as well as the 1-year CIP deviation to match the maturity used by Jiang et al. (2021a). As we discuss below, we cannot extend the same methodology to 10-year CIP deviations since they violate the required assumption of stationarity and independence of CIP from exchange rate premia. The coefficient of interest, χ_k^*, can be estimated from the data using:

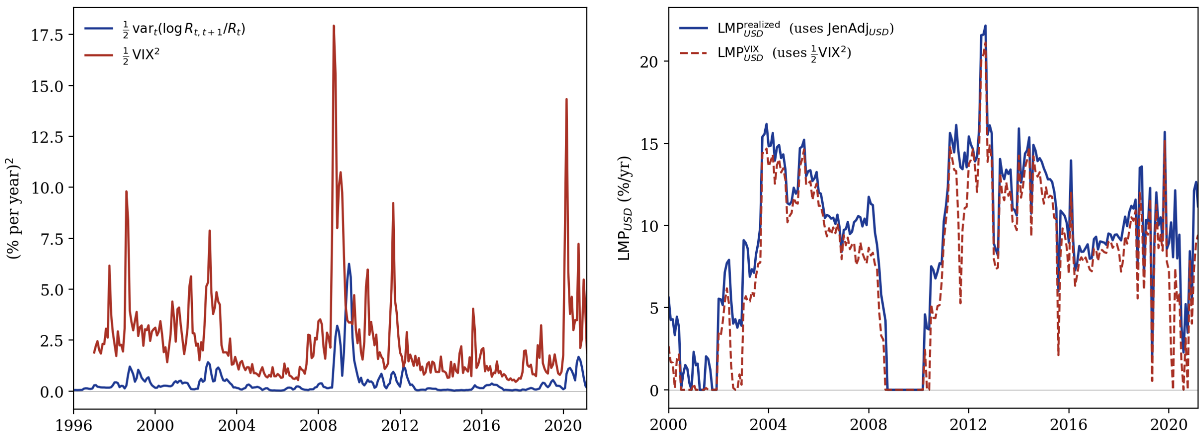
$$\chi_k^* = 1 + \frac{1}{1 - \left[\alpha_1^{(k)}\right]^{h_k}} \frac{1}{\delta_1^{e(k)}} \tag{B.2}$$

[33]The correction is small and largely common across countries: in the U.S.–AE differential it has a standard deviation of 0.20p.p. and a correlation with $\mathcal{D}PermRisk_t$ of −0.03. The long-run coefficient in Table 2 column (1) ranges from −1.31 to −1.38 across alternative horizon and scaling conventions for this term—including omitting it entirely—against a standard error of 0.27.

Figure B.5: Variance Correction and U.S. Permanent Risk: Realized Variance versus $\frac{1}{2}\text{VIX}^2$

(a) Variance correction: $\frac{1}{2}\text{VIX}^2$ vs. $\frac{1}{2}\,\text{var}_t(\log R_{t,t+1}/R_t)$

(b) U.S. permanent risk $L_t(M^{\mathbb{P}}_{t+1})$ under each variance correction



Note. Panel B.5a plots the two candidate variance-correction series considered in the construction of U.S. permanent risk: the option-implied $\frac{1}{2}\text{VIX}^2 = \text{VIX}^2/200$ (red), and the realized $\frac{1}{2}\,\text{var}_t(\log R_{t,t+1}/R_t)$ (navy), constructed from the monthly sample variance of daily 6-month log equity excess returns divided by two, following Martin (2017); the wedge between them is the variance risk premium. Panel B.5b plots U.S. permanent risk $\mathcal{L}_t(M^{\mathbb{P}}_{t,t+1})_{USD,t} = \max\big(\text{ERP}_{USD,t} - V_t - \text{TPcorr}_{USD,t},\, 0\big)$ for each choice of V_t: the realized half-variance (navy) and $\frac{1}{2}\text{VIX}^2$ (red, dashed). Because $\frac{1}{2}\text{VIX}^2$ sits systematically above the realized half-variance, $\mathcal{L}_t(M^{\mathbb{P}}_{t,t+1})^{\text{VIX}}_{USD}$ is uniformly weakly below $\mathcal{L}_t(M^{\mathbb{P}}_{t,t+1})^{\text{realized}}_{USD}$ ($\rho = 0.97$ over 2000:01–2021:03). Sample: 1996:01–2021:03.

where h_k denotes the bond maturity in quarters (i.e., $h_k = 4$ for the 1-year and $h_k = 2$ for the 6-month maturity), α_1 is the persistence of an AR(1) process for the average U.S. CIP deviation across currencies, and δ^e_1 is the marginal effect of innovations to CIP deviations on exchange rate movements. The intuition for this formula comes from solving Proposition 1 forward for the exchange rate level:

$$
\begin{aligned}
e_t = {} & \sum_{h=0}^{\infty} \mathbb{E}_t \left[r^{(k)*}_{t+hk} - r^{(k)}_{t+hk} \right] - \sum_{h=0}^{\infty} \mathbb{E}_t \left[\theta^{F,H(k)}_{t+hk} - \theta^{F,F(k)}_{t+hk} \right] \\
& - \sum_{h=0}^{\infty} \mathbb{E}_t \left[\mathcal{L}_{t+hk}\big(M_{t+hk,t+(h+1)k}\big) - \mathcal{L}_{t+hk}\big(M^{*}_{t+hk,t+(h+1)k}\big) \right] + \bar{e},
\end{aligned}
\tag{B.3}
$$

where $\theta^{F,H(k)}_{t+h} - \theta^{F,F(k)}_{t+h} = CIP^{(k)}_{t+h}/(1-\chi^*_k)$. That is, the future path of convenience yields must move 1-for-1 with exchange rates, which the equation above ensures. Importantly, since $\theta^{F,H(10Y)}_{t+h} - \theta^{F,F(10Y)}_{t+h}$ is non-stationary, its infinite sum is not well defined, which precludes extending the mapping from CIP to convenience yields at long maturities. As discussed in the main text, however, this is not an issue for our error correction model as long as the scale factor is linear and constant. Nonetheless, we show below that there are important differences along the term structure of CIP deviations. The coefficient α_1 is estimated from a quarterly AR(1) process for the average U.S. CIP deviation across the six remaining AE currencies $\overline{CIP}_t^{(k)}$, for a maturity k:

$$\overline{CIP}_{t+3}^{(k)} = \alpha_1 \overline{CIP}_t^{(k)} + \nu_t \tag{B.4}$$

Table B.4 reports the results.

Table B.4: Autocorrelation in CIP Deviations

Vars	$\overline{CIP}_{t+3}^{(6M)}$	$\overline{CIP}_{t+3}^{(1Y)}$
$\overline{CIP}_t^{(6M)}$	0.68***	
	(0.05)	
$\overline{CIP}_t^{(1Y)}$		0.80***
		(0.04)
Observations	230	230
Within R^2	0.46	0.64

The estimation of δ_1^e is in two steps. First, we construct quarterly innovations to the CIP deviation, $\Delta\widetilde{CIP}_t^{(k)}$, as the residual from estimating:

$$\Delta\overline{CIP}_{t+3}^{(k)} = \gamma_0 + \gamma_1 \overline{CIP}_t^{(k)} + \gamma_2 \left(\overline{r_t^{*,(k)} - r_t^{(k)}}\right) + \Delta\widetilde{CIP}_t^{(k)} \tag{B.5}$$

The results from this regression are reported in Table B.5.

Table B.5: Constructing CIP Innovations

Vars	$\Delta\overline{CIP}_{t+3}^{(6M)}$	$\Delta\overline{CIP}_{t+3}^{(1Y)}$
$\overline{CIP}_t^{(6M)}$	-0.59***	
	(0.06)	
$\overline{r_t^{*,(6M)} - r_t^{(6M)}}$	227	
	(138)	
$\overline{CIP}_t^{(1Y)}$		-0.44***
		(0.05)
$\overline{r_t^{*,(1Y)} - r_t^{(1Y)}}$		170
		(83)
Observations	230	230
R^2	0.30	0.23

In the second step, we estimate δ_1^e by regressing quarterly exchange rate movements on these innovations:

$$\Delta \bar{e}_{t+3} = \delta_1^e \Delta \widetilde{CIP}_t^{(k)} + \varepsilon_t \tag{B.6}$$

Table B.6 reports the results.

Table B.6: CIP Innovations and Exchange Rate Dynamics

Vars	$\Delta \bar{e}_{t+3}$	$\Delta \bar{e}_{t+3}$
$\Delta \widetilde{CIP}_t^{(6M)}$	-5.71***	
	(1.15)	
$\Delta \widetilde{CIP}_t^{(1Y)}$		-13.8***
		(1.85)
Observations	230	230
Within R^2	0.10	0.19

Using these estimates α_1 and δ_1^e, we find $\chi^{(1Y)} = 0.88$, which is similar to the value found by Jiang et al. (2021a) of $\chi^{(1Y)} = 0.9$. The value found for the 6-month maturity is $\chi^{(6M)} = 0.68$, which suggests that for longer-maturity Treasuries, a smaller share of their total convenience represents compensation above their currency denomination, consistent with distinct pricing dynamics across maturities.

B.4 Within-Country Convenience Yields

We measure within-country convenience yields according to:

$$\theta_t^{H,H(k)} := r_{irs,t}^{(k)} - r_t^{(k)} \tag{B.7}$$

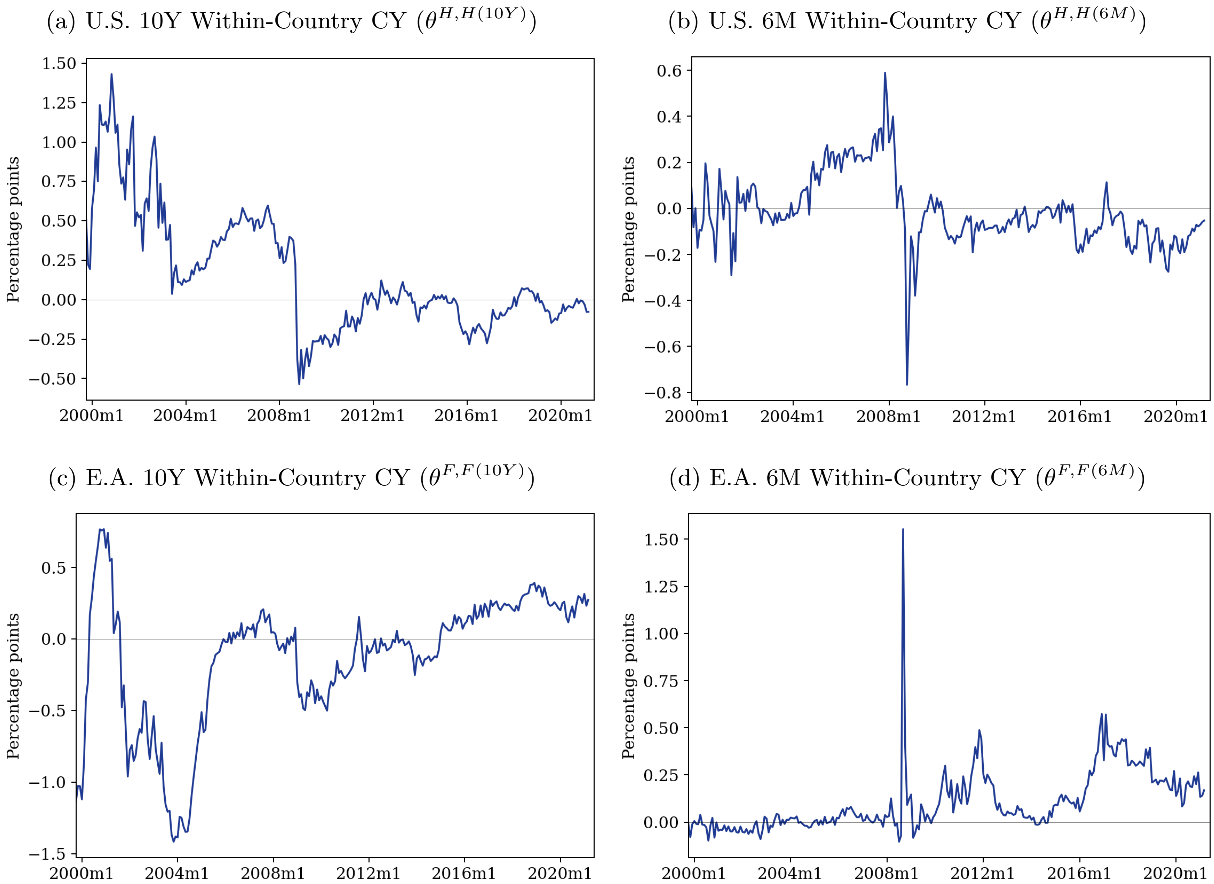
$$\theta_t^{F,F(k)} := r_{irs,t}^{*(k)} - r_t^{*(k)} \tag{B.8}$$

where $r_{irs,t}^{(k)}$ and $r_{irs,t}^{*(k)}$ denote the Home and Foreign log k-period interest-rate-swap rate, respectively. Note that, due to data limitations, we can only construct Foreign within-country convenience yields for the euro-area.

Figure B.6 plots U.S. within-country convenience yield for the 10-year (panel (a)) and 6-month (panel (b)) tenors. Short maturity yields were rising in the build-up to the GFC, but have since averaged just below zero. There are also distinct spikes in 2008 associated with short term funding frictions. In contrast, for long maturities, there is a clear downward trend post 2000, albeit less pronounced than its cross-country counterpart. Therefore, the fading 'specialness' of

U.S. treasuries to Foreign investors is, to an extent, matched by domestic investors in the U.S. Panels (c) and (d) plot within-country convenience yields for the euro area. For both tenors, convenience yields display the opposite dynamics to the U.S. and have been rising over time, with the trend again more pronounced for the 10-year tenor, even focusing only at the post GFC sample (which avoids any data issues and does not rely on back-filling with shorter maturity OIS rates).

Figure B.6: Long- and Short-Maturity Within-Country U.S. and E.A. Convenience Yields



(a) U.S. 10Y Within-Country CY ($\theta^{H,H(10Y)}$)

(b) U.S. 6M Within-Country CY ($\theta^{H,H(6M)}$)

(c) E.A. 10Y Within-Country CY ($\theta^{F,F(10Y)}$)

(d) E.A. 6M Within-Country CY ($\theta^{F,F(6M)}$)

Note. Panels B.6a, B.6b, B.6c and B.6d display time series of long-maturity and short-maturity within-country U.S. Treasury and German Bund convenience yields. The convenience yields are calculated as the difference between interest-rate swap rates and the corresponding-maturity zero-coupon government bond yield, as described in equations (B.7) and (B.8). The sample runs from 1999:10 to 2021:03 based on the availability of data used to estimate within-country convenience yields and CIP deviations.

B.5 Alternative Assumption on Buyback Growth and Growth-Optimal Portfolio

In our headline construction the expected risk premium ERP_t depends on expected total payouts to shareholders — dividends plus net buybacks — and therefore on an assumption about expected buyback growth. The baseline imposes a constant payout ratio, so expected buyback growth equals expected dividend growth, $\widehat{g}_t^{BB} = \widehat{g}_t^D$; the predicted dividend-growth rate $\widehat{g}_t^D$, obtained from our headline dividend-growth regression, is applied uniformly to total payout (dividends plus net buybacks) in equation (2). The alternative treats expected buyback growth as zero — buybacks are viewed as "unexpected" from the standpoint of a date-t forecaster — and applies $\widehat{g}_t^D$ only to the dividend component of total payout. Linearising equation (2) around small payout growth, this is equivalent, to first order, to rescaling the payout-growth term by the dividend share of total payout: the effective predictable payout-growth piece in ERP_t becomes $(\frac{D_t}{P_t}/(\frac{D_t}{P_t}+\frac{B_t}{P_t}))\widehat{g}_t^D$ rather than $\widehat{g}_t^D$, and the ERP_t series correspondingly down-weights the forecast component. Table B.7 re-estimates the headline error-correction model (Table 2, column (1)) using a permanent-risk series built under this alternative. The long-run coefficient on relative permanent risk is essentially unchanged (-1.458^{***} versus -1.330^{***} in the baseline), the Engle–Granger residual ADF continues to reject the unit-root null at the 1% level, and the short-run disequilibrium-adjustment coefficient $\widehat{\gamma}$ remains highly significant. The headline conclusions are robust to the buyback-growth assumption.

Column (3) constructs the growth-optimal portfolio following Bansal and Lehmann (1997), using the baseline buyback growth assumption. In principle, this is the portfolio for which Lemma 2 holds with equality, as detailed in Appendix A.10. The leverage on equity is calculated from full-sample moments as follows

$$\phi_z = \log \mathbb{E}[R_z^{eq}/R_z]/\sigma^2(\log R_z^{eq}/R_z),$$

for each country z. The results are largely unchanged, however, the coefficients change to reflect the new scaling of equity returns.[34]

[34] The levered portfolio is roughly three times as volatile as raw equity, so the smaller coefficient reflects units rather than a weaker relationship: a one-standard-deviation increase in relative permanent risk is associated with a 7.1b.p. fall in long-maturity CIP deviations under the growth-optimal construction, against 7.5b.p. in the baseline and 6.5b.p. under the alternative buyback assumption.

Table B.7: Headline Error-Correction Model: Dividend Construction and Growth-Optimal Portfolio.

	(1) Baseline	(2) Unexpected Buybacks	(3) GOP
Panel A: Long-Run Adjustment			
$\mathcal{D}PermRisk_t$	-1.330^{***}	-1.458^{***}	-0.454^{***}
	(0.270)	(0.423)	(0.086)
$rx_{t+1}^{CT(10Y)}$	0.203^{**}	0.225^{*}	0.174^{**}
	(0.098)	(0.115)	(0.089)
Deterministic Trend	-0.164^{***}	-0.161^{***}	-0.177^{***}
	(0.017)	(0.020)	(0.015)
R^2	0.780	0.748	0.792
Engle–Granger ADF	-5.47^{***}	-4.95^{***}	-5.63^{***}
Panel B: Short-Run Adjustment			
$\Delta\mathcal{D}PermRisk_t$	-0.392^{***}	-0.466^{**}	-0.138^{***}
	(0.143)	(0.235)	(0.052)
$\Delta rx_{t+1}^{CT(10Y)}$	0.115	0.127	0.100
	(0.075)	(0.076)	(0.073)
Diseq. Adjustment $\hat{\gamma}$	-0.235^{***}	-0.205^{***}	-0.251^{***}
	(0.043)	(0.039)	(0.046)
Adj. R^2 (ECM)	0.119	0.102	0.127
# Observations (Panel A / B)	246 / 254	246 / 254	246 / 254

Notes: Column 1: baseline (constant payout ratio). Column 2 assumes buyback growth is unexpected — a first-order Taylor approximation of equation (2) where payout-growth corresponds only to the dividend share. Column 3: growth-optimal portfolio $R_z^g = \phi_z R_z^{eq} + (1 - \phi_z)R_z$, with ϕ from full-sample moments and country-specific leverages $\phi_{USD} = 3.81$, $\phi_{AUD} = 2.77$, $\phi_{CAD} = 2.55$, $\phi_{CHF} = 2.81$, $\phi_{EUR} = 1.30$, $\phi_{GBP} = 1.91$, $\phi_{JPY} = 1.40$. Estimation as in Table 2: DOLS long run, error correction on the implied disequilibrium term, Engle–Granger ADF on the static first-stage residual. Newey–West (6 lags) standard errors. Sample: 2000:01–2021:03.

B.6 Cross-Country and Sub-Sample Stability of the Long-Run Relationship

In this appendix, we demonstrate the robustness of our main results by analysing the full panel of AE variables, *vis-à-vis* the U.S., as opposed to only their cross-sectional average, as well as investigate sub-sample stability. For each foreign currency, we estimate the bilateral analogue of the headline long-run regression of Table 2 Column (1), regressing the long-maturity CIP deviation against the foreign currency on the relative permanent-risk differential, the realized 10-year dollar-trade return, and a deterministic trend, estimated by dynamic OLS as in the headline specification. We also report the equally-weighted aggregates EW6 (the full AE-ex-U.S. panel, identical to our headline) and EW4 (excluding CHF and JPY, the two safe-haven currencies), and the Mean-Group estimator (i.e. the mean of the bilateral β) for both aggregations. The sample is split into "Full" (2000:01–2021:03), "Pre-GFC" (2000:01–2008:06), and "Post-GFC" (2008:07–2021:03).

Three patterns emerge. First, the EW4 aggregate slope is remarkably stable across the three subperiods (-1.45^{***}, -1.10^{***}, -1.44^{***}, respectively) and cointegrates at the 1% level in the full and pre-GFC samples. Although the post-GFC residual ADF (-3.44) does not reject, this may reflect low power in the shorter sample, given that the slope itself remains stable and significant. The pooled EW6 aggregate slope is similarly stable for the full and pre-GFC subperiods (-1.33^{***} and -1.36^{***}, both cointegrating at the 1% level) but weakens in the post-GFC subperiod. Second, the post-GFC weakening of EW6 is driven by a sign flip in CHF and JPY, the two safe-haven currencies: bilateral slopes that are significantly negative pre-GFC (-1.68^{***} and -1.73^{***}, respectively) turn significantly positive post-GFC (1.76^{***} and 1.81^{***}). The Swiss results should be read with caution: CHF has the least well-identified dividend-growth forecast in the panel, with an adjusted R^2 of 0.11 against 0.46 for the U.S. (Table B.3).[35] Once these two currencies are excluded, the EW4 aggregate recovers its full-sample magnitude post-2008 almost exactly (-1.44 against -1.45 over the full sample), indicating that the apparent EW6 break is driven by safe-haven dynamics rather than a broad breakdown of the underlying relationship across all currencies. Third, the Mean-Group estimator corroborates the pooled aggregates: $\hat{\beta}_{MG}^{EW6}$ tracks the pooled EW6 slope qualitatively but is smaller in magnitude over the full and post-GFC samples (a consequence of the substantial cross-currency dispersion), whereas $\hat{\beta}_{MG}^{EW4}$ remains significantly negative throughout.

[35] This may reflect the concentration of the SMI in a small number of large-cap payers—roughly half its weight sits in three firms—which makes six-month dividend growth lumpy and hard to predict. Swiss distributions are also understated: Nestlé and Novartis at least run substantial buyback programmes for which we have no data. Neither issue affects our aggregate results: replacing the Swiss buyback yield with its euro-area analogue, or re-estimating the Swiss forecast with euro-area GDP growth expectations, leaves the long-run coefficient in Table 2 at -1.33 in both cases.

Table B.8: Bilateral Long-Run Permanent-Risk Slope by Currency and Subsample.

	Full 2000:01–2021:03	Pre-GFC 2000:01–2008:06	Post-GFC 2008:07–2021:03
AUD	−2.390***	−1.193***	−1.478**
	(0.731)	(0.324)	(0.746)
CAD	−2.755***	−2.259***	−2.405***
	(0.326)	(0.354)	(0.742)
CHF	−0.127	−1.675***	1.762***
	(0.634)	(0.480)	(0.454)
EUR	−0.625***	−0.907***	−0.528*
	(0.165)	(0.163)	(0.300)
GBP	−0.458	−0.096	−0.347
	(0.507)	(0.178)	(0.665)
JPY	0.563	−1.726***	1.808***
	(0.805)	(0.227)	(0.670)
EW6 aggregate	−1.330***	−1.358***	−0.341
	(0.270)	(0.225)	(0.285)
EG-ADF	−5.47***	−6.05***	−3.85*
Mean Group (EW6)	−0.965*	−1.309***	−0.198
	(0.537)	(0.308)	(0.695)
EW4 aggregate	−1.447***	−1.098***	−1.442***
(no CHF, JPY)	(0.257)	(0.227)	(0.423)
EG-ADF	−5.05***	−5.56***	−3.44
Mean Group (EW4)	−1.557***	−1.114**	−1.189**
(no CHF, JPY)	(0.592)	(0.447)	(0.475)
# Observations	246	93	144

Notes: Long-run slopes on $\mathcal{D}PermRisk_t^z$ from bilateral DOLS regressions (four leads and lags), with Newey–West HAC standard errors. EW6/EW4 rows use the equally-weighted aggregates (EW4 drops CHF and JPY) and report the Engle and Granger (1987) residual ADF(1) from the static first stage. Mean-Group rows average the bilateral slopes. CIP in basis points, risk in percent points. Stars on slopes use normal critical values; stars on the EG-ADF rows use MacKinnon (2010) critical values.

C Further Segmentation and the Short-Run Disconnect

We assume "short" investors price short debt and "long" investors price all long debt and equities. This segmentation can be due to regulation, taxation or more generally a preferred habitat, over and above the bond- specific convenience yields. This layer of segmentation is over and above that implied by convenience-yields which lie at the centre of our focus. In contrast to convenience yields, this segmentation is common across all long-lived assets, both bonds and equity. For ease of exposition, we focus on the complete spanning limit.

Define the safe investor's $\hat{M}$ and the risky investor's SDF is related as follows $M = \hat{M}e^{\Delta^{R/S}}$. Consequently:

$$\mathcal{L}(M) = \mathcal{L}(\hat{M}) + \frac{1}{2}\sigma_{\Delta}^{2\ R/S}, \tag{C.1}$$

where we assume, for simplicity, conditional log-normality and orthogonality for the $\Delta^{R/S}$-wedge and $\mathbb{E}[\Delta^{R/S}] = 0$. More generally, the wedge can be allowed to be heteroskedastic and the derivations below follow accordingly. Consistent with a high risk premium, *risky* investors have a higher SDF volatility.

The extended framework we present below shows that using equities measures the SDF volatility of *long* investors. Instead, the equilibrium dollar trade returns with short maturity assets reflect the risk differentials for *short* investors—so this condition is mismeasured. Turning to dollar trade returns on portfolios formed with long maturity bonds, the relevant SDF volatility switches to that of the "long" investors which is correctly measured by equities.[36]

The relevant set of Euler equations for safe assets is as follows:

$$\mathbb{E}_t\left[\hat{M}_{t,t+1}\right] R_t = e^{-\theta_t^{H,H(1)}} \tag{C.2}$$

$$\mathbb{E}_t\left[\hat{M}^*_{t,t+1}\right] R^*_t = e^{-\theta_t^{F,F(1)}} \tag{C.3}$$

$$\mathbb{E}_t\left[\hat{M}_{t,t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right] R^*_t = e^{-\theta_t^{H,F(1)}} \tag{C.4}$$

$$\mathbb{E}_t\left[\hat{M}^*_{t,t+1}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right] R_t = e^{-\theta_t^{F,H(1)}} \tag{C.5}$$

[36]Such frictions between asset markets can also explain the divergence between the Treasury basis and AAA or Refcorp spreads.

Since we focus on the complete spanning limit, the exchange rate process is given by:

$$\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = \frac{\hat{M}^*_{t,t+1}}{\hat{M}_{t,t+1}} e^{\theta^{F,H(1)} - \theta^{H,H(1)}} \tag{C.6}$$

and $\theta^{F,H(1)} - \theta^{H,H(1)} = \theta^{F,F(1)} - \theta^{H,F(1)}$. The process can be verified by plugging $\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$ back into the Euler equations.

On the other hand, for *long* investors characterized by $M_{t,t+1}$, they invest in risky portfolios (equities):

$$\mathbb{E}_t \left[M_{t,t+1} R^g_{t,t+1} \right] = 1 \tag{C.7}$$

$$\mathbb{E}_t \left[M^*_{t,t+1} R^{g*}_{t,t+1} \right] = 1 \tag{C.8}$$

and long maturity bonds:

$$\mathbb{E}_t \left[M_{t,t+k} \right] R^{(k)}_t = e^{-\theta^{H,H(k)}_t} \tag{C.9}$$

$$\mathbb{E}_t \left[M^*_{t,t+k} \right] R^{(k)*}_t = e^{-\theta^{F,F(k)}_t} \tag{C.10}$$

$$\mathbb{E}_t \left[M_{t,t+k} \frac{\mathcal{E}_{t+k}}{\mathcal{E}_t} \right] R^{(k)*}_t = e^{-\theta^{H,F(k)}_t} \tag{C.11}$$

$$\mathbb{E}_t \left[M^*_{t,t+k} \frac{\mathcal{E}_t}{\mathcal{E}_{t+k}} \right] R^{(k)}_t = e^{-\theta^{F,H(k)}_t} \tag{C.12}$$

Following Alvarez and Jermann (2005), using the Eulers for long-maturity bonds held to maturity, we can calculate:

$$R^{(\infty)}_{t,t+1} = M^{\mathbb{T}\ -1}_{t,t+1} e^{-\theta^{H,H(\infty)}_{t,t+1}}, \quad R^{(\infty)\ *}_{t,t+1} = M^{*\mathbb{T}\ -1}_{t,t+1} e^{-\theta^{F,F(\infty)}_{t,t+1}} \tag{C.13}$$

Intuitively, to derive this the short bonds were never used, so the holding period return only captures the "long" investor.

Using the Euler for risky assets:

$$\mathcal{L}(M_{t,t+1}) \geq \log \mathbb{E}_t[M_{t+1}] + \mathbb{E}_t \log R^g_{t+1} \tag{C.14}$$

Moreover, we can express:

$$\begin{aligned}\mathcal{L}(M_{t,t+1}) &= \log \mathbb{E}_t[M_{t,t+1}] - \mathbb{E}_t[\log M^{\mathbb{T}}_{t,t+1} M^{\mathbb{P}}_{t,t+1}] \\ &= \log \mathbb{E}_t[M_{t,t+1}] + \mathcal{L}(M^{\mathbb{P}}_{t,t+1}) - \mathbb{E}\left[\log\left(\frac{1}{R^{(\infty)}_{t,t+1}} e^{-\theta^{H,H(\infty)}_{t,t+1}}\right)\right]\end{aligned} \tag{C.15}$$

where the second equality uses $M = M^{\mathbb{P}} M^{\mathbb{T}}$. Combining with (C.14), yields our measurement equation for the "risky" SDF:

$$\mathcal{L}(M^{\mathbb{P}}_{t,t+1}) \geq \mathbb{E} \log \frac{R^g}{R^\infty} - \mathbb{E}_t \theta^{H,H(\infty)}_{t,t+1} \tag{C.16}$$

Similar derivations apply abroad.

Derivations of Short-Maturity Relationship. Next we derive the short-run dollar trade relationship. Taking logs of (C.6) and expectations:

$$\mathbb{E}_t[\Delta e_{t+1}] = \mathbb{E}_t[\hat{m}^*_{t,t+1}] - \mathbb{E}_t[\hat{m}_{t,t+1}] + \theta^{F,H(1)}_t - \theta^{H,H(1)}_t \tag{C.17}$$

Subbing out using the short-run ("safe") Eulers yields Proposition 1:

$$\underbrace{\mathbb{E}_t[\Delta e_{t+1}] + r^*_t - r_t}_{\mathbb{E}_t[rx^{FX}_{t+1}]} = \mathcal{L}(M_{t,t+1}) - \mathcal{L}(M^*_{t,t+1}) - \frac{1}{2}[\sigma^2_\Delta - \sigma^{*2}_\Delta] + \theta^{F,H(1)} - \theta^{F,F(1)} \tag{C.18}$$

where we use (C.1). Intuitively, we have no measure of the "safe" SDF volatility, so the short-run equilibrium relationship has an error term $-\frac{1}{2}[\sigma^2_\Delta - \sigma^{*2}_\Delta]$.

Derivations of Long-Maturity Relationship. To derive the long run relationship, we use (C.15):

$$\mathcal{L}(M_{t,t+1}) = \log \underbrace{E_t[\hat{M}_{t,t+1}]}_{\frac{1}{R_t} e^{-\theta^{H,H(1)}_t}} + \frac{1}{2}\sigma^2_\Delta + \mathcal{L}(M^{\mathbb{P}}_{t,t+1}) - \mathbb{E}\left[\log\left(\frac{1}{R^{(\infty)}_{t,t+1}} e^{-\theta^{H,H(\infty)}_{t,t+1}}\right)\right] \tag{C.19}$$

Substituting this into Proposition 1 and using the identity $\theta^{F,H(1)}_t - \theta^{F,F(1)}_t = \theta^{H,H(1)}_t - \theta^{H,F(1)}_t$

gives, after collecting:

$$\mathbb{E}_t\big[\mathrm{rx}^{CT,(\infty)}_{t+1}\big] \;=\; \mathcal{L}_t(M^{\mathbb{P}}_{t,t+1}) - \mathcal{L}_t(M^{\mathbb{P}*}_{t,t+1}) + \mathbb{E}_t\big[\theta^{H,H(\infty)}_{t,t+1} - \theta^{F,F(\infty)}_{t,t+1}\big] + \theta^{F,H(1)}_t - \theta^{H,H(1)}_t, \quad \text{(C.20)}$$

where $\mathbb{E}_t[\mathrm{rx}^{CT,(\infty)}_{t+1}] = \mathbb{E}_t[\mathrm{rx}^{FX}_{t+1}] - (\mathbb{E}_t[\mathrm{rx}^{(\infty)}_{t+1}] - \mathbb{E}_t[\mathrm{rx}^{(\infty)*}_{t+1}])$.

Derivation of Term Structure. Applying Lemma 3 in the limit $k = \infty$:

$$\mathbb{E}_t[\theta^{H,H(\infty)}_{t,t+1}] \;=\; \mathbb{E}_t[\theta^{F,H(\infty)}_{t,t+1}] - \big(\theta^{F,H(1)}_t - \theta^{H,H(1)}_t\big) - \Phi^{H(\infty)}_t. \quad \text{(C.21)}$$

Under complete spanning, $\Phi^{H(\infty)}_t$ reduces to $-\mathcal{L}_t\big(e^{-(\theta^{F,H(\infty)}_{t+1}-\theta^{H,H(\infty)}_{t+1})}\big) - \mathcal{C}_t\big(M_{t,t+1},\, e^{-(\theta^{F,H(\infty)}_{t+1}-\theta^{H,H(\infty)}_{t+1})}\big)$, which is non-zero because $\theta^{F,H(\infty)}_{t+1} - \theta^{H,H(\infty)}_{t+1}$ is $(t+1)$-measurable.

Substituting (C.21) into (C.20) and cancelling the one-period basis $\theta^{F,H(1)}_t - \theta^{H,H(1)}_t$:

$$\mathbb{E}_t\big[\mathrm{rx}^{CT,(\infty)}_{t+1}\big] \;=\; \mathcal{L}_t(M^{\mathbb{P}}_{t,t+1}) - \mathcal{L}_t(M^{\mathbb{P}*}_{t,t+1}) + \mathbb{E}_t\big[\theta^{F,H(\infty)}_{t,t+1} - \theta^{F,F(\infty)}_{t,t+1}\big] - \Phi^{H(\infty)}_t\Big|_{\mathrm{cs}}, \quad \text{(C.22)}$$

which is Proposition 2 evaluated at the complete-spanning limit, with $\Omega^{(\infty)}_t = \Phi^{H(\infty)}_t|_{\mathrm{cs}} + \Omega^{(1)}_t = \Phi^{H(\infty)}_t|_{\mathrm{cs}}$ since $\Omega^{(1)}_t \to 0$ under complete spanning.

Critically, in contrast to Proposition 1, the relevant SDF volatility for the long-maturity relationship is correctly measured using equities and long bonds. The short-maturity mismeasurement $-\frac{1}{2}[\sigma^2_\Delta - \sigma^{*2}_\Delta]$ is absent at long maturities.

Table C.1 below illustrates that the short-maturity basis continues to co-move with funding and intermediary conditions: it loads positively on the VIX and, more strongly and robustly, on intermediary leverage, while the dollar-trade return is insignificant. Since variables are stationary, we run the regression in levels. Our findings suggest that short-maturity CIP deviations (and the segmentation wedge we emphasize $\Delta^{R/S(*)}$) reflects balance-sheet and funding frictions, rather than the risk channel that governs long maturities.

Table C.1: Short-Maturity CIP Deviations and Funding Conditions (Levels)

Dep. Var.: $CIP_t^{(6M)}$	(1)	(2)
rx_{t+1}^{FX}	-0.321	-0.299
	(0.273)	(0.262)
VIX_t	1.089*	0.813
	(0.606)	(0.564)
Lev_t		0.783***
		(0.247)
Adjusted R^2	0.144	0.170

Notes: Regressions of the short-maturity (6-month) CIP deviation (in basis points), in levels, on the short-maturity dollar-trade return, the VIX, and intermediary leverage Lev_t (the inverse of the He et al. (2017) capital ratio). Sample: 2000:01–2021:03. Newey and West (1987) standard errors with 6 lags in parentheses; ***, **, * denote $p < 0.01, 0.05, 0.10$.